Memo 108

Determining the Specification of an Aperture Array for Cosmological Surveys

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In this memo we discuss the required specification of an aperture array for the SKA which is capable of performing the cosmological surveys needed to reach the objectives of the SKA key science. We concentrate on the frequency range 300MHz – 1GHz: we only briefly consider the requirements for an "Epoch of re-ionization" array. The most demanding requirements are set by the deep HI redshift surveys. A key strength of aperture arrays is the ability to change the number of beams which are formed and processed in the beam former as a function of frequency and hence redshift. Using the aperture array we are therefore able to change the instantaneous field of view and hence survey speed as a function of redshift. A second, equally important design consideration is whether the aperture array either fully samples the incoming wave front at all frequencies, or is sparse over some frequency range. We take a, possibly conservative, view that a fully sampled aperture array will provide the best opportunity for obtaining very high dynamic range for the frequency range above 500MHz where the sky brightness is relatively low. A sparse aperture array has the advantage of an increasing effective area with decreasing frequency which can be used to help combat the increasing sky brightness at low frequency. We consider an overall aperture array design combining a sparse aperture array at lower frequencies and a fully sampled aperture array at higher frequencies taking technical constraints, in particular for processing, from the work of the SKADS project on the "Benchmark Scenario". We then put forward a possible specification for an aperture array which would be able to deliver the key science goals of an HI survey and how these specifications match the requirements of other key-science aims especially continuum surveys. It should be noted that this is not a fully optimised design, which is expected to be determined by detailed simulations and cost modelling to deliver the overall SKA science case. We examine in more detail two possible experiments that could be performed with such an instrument.

We believe the required performance for the SKA is both achievable and affordable. The analysis discussed here forms the basis for the proposed aperture-array specification presented by the SKA specification Tiger Team in 2007, now published as SKA Memo 100, Preliminary Specifications for

the Square Kilometre Array (Schilizzi et al. 2007), hereafter referred to as Memo 100.

1. Introduction

A common requirement of much of the key science case for the SKA – especially at frequencies of L-band and below – is high survey speed. Of these the proposed large-area HI-redshift surveys provide the most demanding requirements for survey speed (e.g. Abdalla & Rawlings 2005, Jackson 2006). An Aperture Array (AA) provides the most attractive technology to achieve these extremely high survey speeds, provided the aperture arrays can be realised at an affordable cost. In this memo we discuss the required specification of an aperture array for the SKA which is capable of performing cosmological surveys of this type. We concentrate on an approximate frequency range of 300MHz – 1GHz: we do not consider here the explicit science requirements for the aperture array below 300 MHz for which the main science driver is studying the epoch of re-ionization (EoR).

A key strength of aperture arrays is the ability to change the number of beams which are formed and processed in the beam former as a function of frequency and hence redshift. Using the aperture array we are therefore able to change the instantaneous field of view and hence survey speed as a function of redshift. A second, equally important design consideration is whether the aperture array either fully samples the incoming wave front at all frequencies, or is sparse over some frequency range (Braun & van Cappellen 2006). We take a, possibly conservative, view that a fully sampled aperture array will provide the best opportunity for obtaining very high dynamic range for the frequency range above 500MHz where the sky brightness is relatively low. A sparse aperture array has the advantage of an increasing effective area with decreasing frequency which can be used to help combat the increasing sky brightness at low frequency.

We consider an overall aperture array design which combines a sparse aperture array at lower frequencies and a fully sampled aperture array at higher frequencies; other aspects of the design closely follow the SKADS "Benchmark Scenario" (Alexander et al. 2007, Bolton et al. 2008). The suggested specifications of such an array are determined below by considering the demands of the HI experiment.

2. Assumptions for the AA design

In this section we outline the assumptions we have made concerning the possible design of the AA covering a frequency range from 70 MHz to 1GHz. These assumptions are important for the derivation of the performance specifications; there is no attempt here to prove these statements which is not the purpose of this memo. More details are given in Alexander et al. (2007) and in Memo 100. We assume that to cover this frequency range the AA consists of at least two and possibly three element types. The AA is considered to be a mix of:

- An AA which is sparse over most (if not all) of its operational frequency range; for this array we make the approximation that the effective area of each element scales as $\lambda^2/4$;
- An AA which is fully-filled over a range of frequencies where the wave front is fully sampled at the Nyquist rate.

Furthermore we will make the following assumptions about the performance of the aperture array:

(a) A fully sampled AA, since it measures the incoming wave front at the Nyquist rate, can by using appropriate processing yield the best controlled aperture response function for a beam. For a large AA station we have complete real-time dynamic control over the complex appodization function across the AA, we are able to calibrate on an element-by-element level to remove systematic effects.

(b) When the AA is sparse we no longer have the ability to fully reconstruct the incoming wave front. We will take the pessimistic view that this will limit the achievable dynamic range for continuum imaging. A discussion of how higher dynamic range may be achieved by considering the form of the cross-power beam is given in Braun and van Cappellen (2006). Further, very detailed analysis is required to determine precisely how well a sparse array can perform in continuum observations. However, we expect to have excellent spectral dynamic range by direct digitisation followed by a high-accuracy poly-phase filter. Therefore we will assume that the sparse AA will be able to reach the required dynamic range specification for the HI imaging and EoR key science.

Other advantages and constraints for the AA which are particularly relevant for this discussion include:

- The survey speed which can be achieved by the AA is limited by our ability to process and transport data – firstly in the tile or station beam former and secondly by data processing post the correlator. By concentrating the AA in a relatively small number of large stations we can reduce the cost (computational and monetary) of the post-correlator processing. The AA survey speed is then limited by the station level DSP and the wide-area data rate achievable to the correlator.
- The instantaneous FoV of each element of the AA is at least 0.6π sr assuming a maximum scan-angle of $\pi/4$.
- The ability to use sparse elements leads to an AA station which has increasing effective area with increasing λ (scaling approximately as λ^2) – this property is essential to help get the sensitivity required at low frequencies when the sky temperature dominates the system temperature (T_{sys}) of the telescope.
- Multiple disjoint fields of view which are controllable dynamically are possible with very fast (almost instantaneous) slew speed.
- Since the AA does not physically track, the sensitivity of the AA changes as we scan away from the zenith – the two main contributions being the geometrical projection of the effective area of the array and the (embedded) element response on the sky. In the analysis below we consider the mean A_{eff}/T_{sys} – we can do

better than this for observations close to the zenith, but the AA is optimised as a survey instrument and therefore we do not wish to impose this additional constraint.

We now consider the cost scaling laws appropriate for an AA – a good approximation (valid for large element counts) is that the AA-specific cost scales as the number of elements; the cost per element includes the element itself plus its share of the signal path to the correlator. With this approximation the elements can be distributed between stations at will (within reason) for the same cost – this is equivalent to the situation where dish cost scales simply as $D^2 \propto A_{eff}$.

For an AA consisting of N_s stations each with N_{es} elements the total element count and hence cost scales as $N_e = N_{es} N_s$. For a fully sampled AA, the effective area is $\sim N_e d^2$, where d is the element separation ($\lambda < \sim 2d$); for a sparse AA the equivalent approximate result is $\sim N_e \lambda^2/4$. These results neglect element coupling and variations of A_{eff} with scan direction, but give the correct scaling behaviour. A conservative design is adopted in which the mid-frequency AA is taken to be a fully-sampled AA for best possible control of systematics and side-lobe levels in order to achieve maximum dynamic range. In this case, the station beam is given approximately by

$$\Omega_b \sim \frac{\lambda^2}{D^2} \sim \frac{\lambda^2}{\mathrm{N_{es}} d^2}$$

The number of beams required to produce a total instantaneous Field of View of the AA of Ω_F is:

$$N_b = \frac{\Omega_F}{\Omega_b} \sim \frac{\Omega_F \mathrm{N_{es}} d^2}{\lambda^2} \propto \Omega_F \mathrm{N_{es}} d^2$$

This is a function of frequency across the band – for an AA Ω_F itself can also be chosen to be a function of frequency. However, it is easy to show that this scaling for the mean number of beams averaged over the observing band always holds; to simplify the analysis the mean number of beams across the band is used below. The number of beams that can be processed from the AA will be limited by either the ability to transmit data to the correlator, or the capacity of the correlator, or the capacity of the post-correlator processor. The data rate from the correlator scales as

$$R_c \propto \mathrm{N_{ch}} \mathrm{N_s}^2 \mathrm{N_b} \propto \mathrm{N_{ch}} \mathrm{N_s}^2 \Omega_F \frac{\mathrm{N_e}}{\mathrm{N_s}} d^2 .$$

Here N_{ch} is the number of frequency channels.

Any realisation of the SKA will use dishes at the higher frequencies – specifically we consider those realisations discussed in Memo 100. The equivalent scaling for dishes is

$N_{ch}\ N_{dish}^2$: the high frequency dishes (with either single pixel feeds or phased array feeds) have significant correlator and post-correlator requirements and these dominate over the requirements of the AA. Hence it can be assumed that R_c is fixed by the high-frequency dishes; provided the correlator is re-configurable this fixes the R_c for the AA. Then the FoV which can be processed scales as $\Omega_F \propto R_c / (N_{ch}\ N_s\ N_e\ d^2)$. The survey speed, as defined in Memo 100 is then given by:

$$\mathrm{FoMSS} \propto B\ R_c\ N_e\ d^2 / (N_{ch}\ N_s\ T_{sys}^2),$$

where B is the bandwidth. The AA cost, for a given survey speed therefore simply scales as

$$\mathrm{COST} \propto N_e \propto \mathrm{FoMSS}\ N_s\ T_{sys}^2 / B\ d^2$$

The cost scales linearly with both survey speed and A_{eff} in this limit when the data rate from the correlator is fixed. For a given FoMSS the number of stations should be minimised and *d* should be maximised. The minimum number of stations is determined by the need to have sufficient aperture-plane coverage for high-dynamic range – this has been fixed at 150 as a minimum value, although the SKADS design assumes 250 stations. R_c is fixed by the high-frequency dish solution in which it is assumed that it is possible to process the data from ~1200 dishes within 5 km (50% of the dishes) with single pixel feeds observing in spectral line mode at L-band.

3. Science requirements and the AA design

We now consider the design of an AA which is optimised for the following key science cases

(i) Probing the dark ages:
  a. mapping the EoR
  b. the first AGN
(ii) Galaxy evolution and dark energy
  a. Evolution of HI
  b. Dark energy

At very low frequency (below about 300 MHz) AA's seem to offer the only cost-effective technology to provide the collecting area needed to get the A_{eff}/T_{sys} required for the EoR experiment whether this is 10000 or 5000 m^2/K – this is also demonstrated by the technology choice for current EoR experiments such as LOFAR.

The science driver for HI (galaxy evolution and dark energy) requires frequency coverage below 1400 MHz. Detecting redshifted HI becomes increasingly challenging as we move out in z to ~ 3-4. To complete a survey to a specified mass limit requires an increased survey speed with increasing z. This translates to a specification for the AA in which we attempt to maximise A_{eff}/T_{sys} at the lower frequencies and have an FoV (number of beams times station beam) increasing faster than λ^2 with wavelength.

Spectral requirements and baselines for these experiments are relatively modest – this is a detection experiment in survey mode so we need to properly sample the HI line and avoid confusion in position-velocity space.

Deep, wide-field, continuum surveys (for example to detect the first AGN) require very high survey speed but also excellent dynamic range. The science drivers require an all-sky survey to of order 1 micro Jansky at ~250 mas resolution or at least enough to avoid confusion. With the AA these experiments can be performed at lower frequencies (~ 500-800 MHz). We adopt a conservative view that to achieve the required dynamic range we require a fully-sampled (element spacing ≤λ/2) AA over some range of frequencies.

For frequencies below f_{sky} ~ 500 MHz the system temperature increases due to the increasing contribution from the sky temperature (see e.g. Medellin 2007). Maintaining a constant sensitivity below this frequency can only be achieved by adding collecting area. A sparse AA helps considerably in this regime since the effective area increases as λ^2 and this can be employed to help offset the effects of increasing sky temperature. To illustrate a possible solution we consider the following outline design illustrated in Figure 1. Sparse collector only needs to cover the frequency band up to f_{Sky} hence a different element type to that used for the fully-sample AA can be employed. The sky noise increases faster than λ^2 with increasing wavelength: below about 250MHz this becomes dominant – this can be offset by adding a third region of the AA to increase the using a third element type.

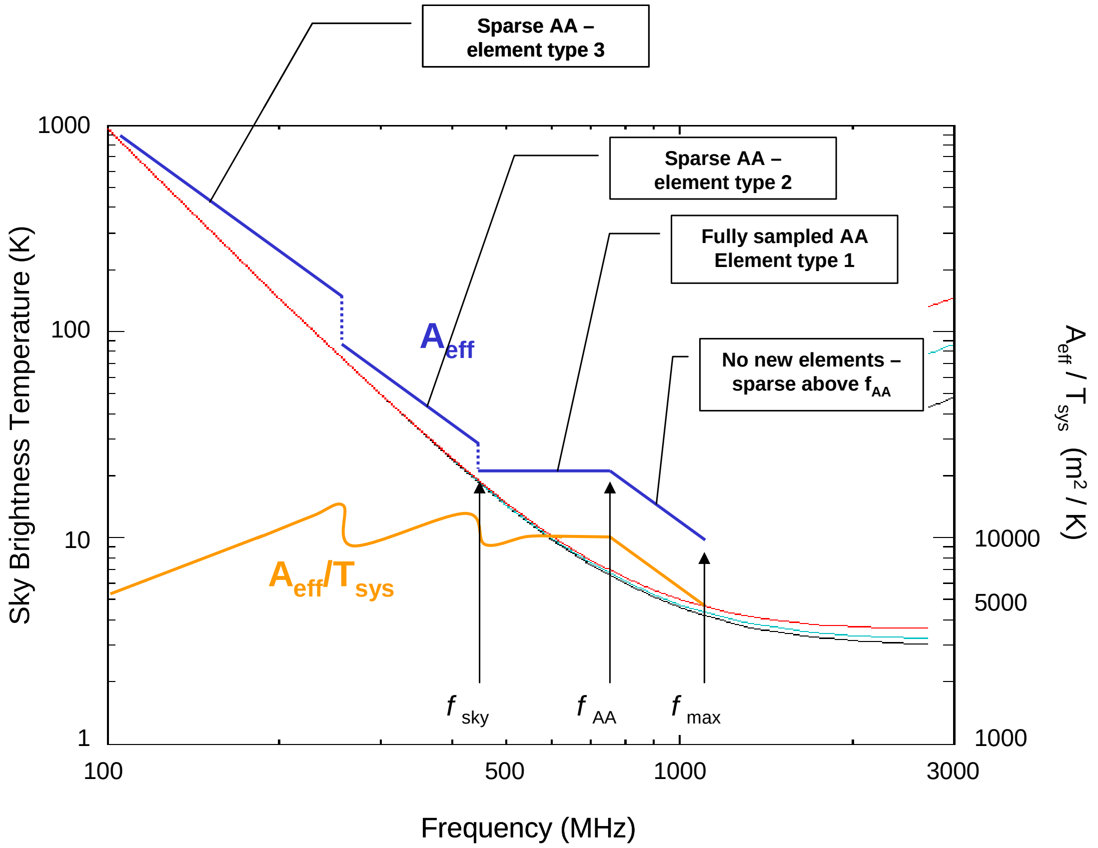

Figure 1: effective area and sensitivity of a possible aperture array design using a mix of fully-sampled and spare aperture array elements.

These considerations lead us to a possible specification for the AA in which we assume A_{eff}/T_{sys} to be approximately constant with frequency below 800 MHz, a fully sampled AA at ~500-800MHz and sparse at lower frequencies. The AA will again be a sparse array above 800 MHz, but will still have excellent survey speed. We now consider if such a specification can achieve the science goals.

For a given flux limit the HI mass within a galaxy which can be detected is:

$$\frac{M_{HI}(z)}{\text{M}_{\text{o}}} = \frac{0.235}{1+z}\frac{D_L^{\;2}(z)}{\text{Mpc}^2}\frac{S_\nu}{\nu\text{Jy}}\frac{V}{\text{kms}^{-1}}$$

Abdalla and Rawlings (2005) argue that $V(\text{z}) = V_0(1+\text{z})^{-1/2}$ and hence the time needed to survey a sky area, Ω with a total processed field of view (which we take to be a function of z) Ω_{F} is:

$$\tau_s \propto \Omega \frac{1}{M(z)^2}\frac{D_L^{\;4}}{(1+z)^3}\left(\frac{T_{sys}}{A_{eff}}\right)^2\frac{1}{\Omega_F(z)}$$

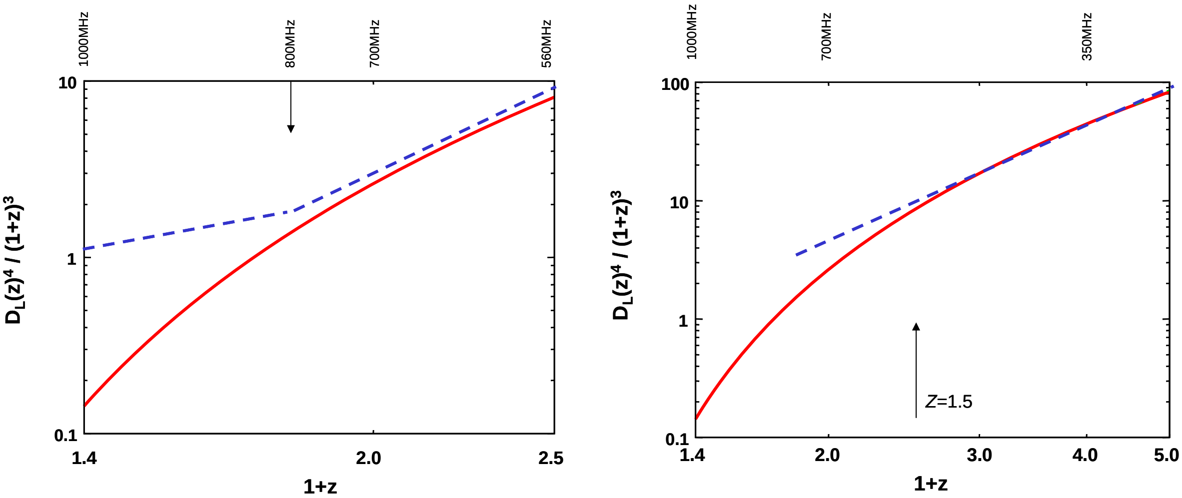

Figure 2: The full line shows the form of the required survey speed in the redshift range (1+z) for two experiments. The left-hand panel refers to the dark energy survey from z 0.4 to 1.5 and right-hand panel to the deep HI survey for a direct measure of curvature.

To detect an M* galaxy at z=0.75 (800 MHz) takes 25 hours of integration for A_{eff}/T_{sys} = 10,000 m^2/K. Now consider an experiment in which we wish to do a survey to some limiting mass *M*(z). To do this we need an effective survey speed which increases as redshift as:

$$t_{SS}(z) \;=\; \Omega_F(z)\left(\frac{A_{eff}}{T_{sys}}\right)^2 \propto \frac{1}{M(z)^2}\frac{{D_L}^4}{(1+z)^3}$$

Here there is no bandwidth appearing in the definition of survey speed, because this is the survey speed at a given redshift. The worst case we will want to do is to survey to a constant mass, then we need a survey speed increasing as:

$$t_{SS}(z) \;\propto \frac{{D_L}^4}{(1+z)^3}$$

This function is sketched approximately in Figure 2 for a concordance cosmology.

3.1 Experiment 1 – Wide-Field Dark-Energy Survey

A key parameter is the data rate which can be processed and sent back to the correlator. A costed design of a potential AA implementation is analysed in the SKADS Design and Costing paper (Alexander et al. 2007) – we assume that the same data rate can be processed as discussed in Memo 93. As an example, for an AA which is fully sampled at 800 MHz we choose the following for the total processed FoV:

$$\Omega_F(\nu) = \begin{cases} A\,(\nu/800)^2 & \nu > 800 \text{ MHz} \\ A\,(\nu/800)^{-5} & \nu < 800 \text{ MHz} \end{cases}$$

The ability to increase the FoV at high frequencies, by processing more beams at these frequencies is a powerful property of the AA. At frequencies below 800 MHz we arrange for the total FoV to increase much faster than λ^2 (again by processing more beams at lower frequencies) in order to maintain the survey speed at increasing redshift. At higher frequencies the AA is sparse and sensitivity drops off. At a fixed data rate this gives the constant A = 130 sq degrees. The form of the resulting survey speed is shown as the dashed line in Figure 2 and closely matches that which is required to do this experiment at low frequencies – at high frequencies the form is chosen to give the same survey speed as a dish with FoV compensating the declining sensitivity.

For a full-sky survey of 20,000 deg^2, assuming A_{eff}/T_{sys} = 6000 m^{2}K^{-1} at 800 MHz this gives a total integration time very close to 1.5 years and hence a likely time-scale for completion of the experiment of 4 to 5 years. The instantaneous FoV processed at 1000 MHz is 200 sq. deg. and 1300 sq.deg. at 500 MHz. Note this experiment is more demanding than that discussed by Abdalla and Rawlings – the behaviour they require is matched out to z = 0.75, and exceeded out to z = 1.5 since all galaxies greater than M* are detected to 10 sigma.

3.2 Experiment 2 – Deep HI Emission for Galaxy Evolution and Measurement of Curvature

Rawlings has recently noted that this will also be important for dark energy as well as for galaxy evolution, since measuring the acoustic oscillations at high-z enables curvature to be measured directly (since Ω_Λ will then be insignificant compared to Ω_m). These are also the redshifts one wants to probe for the galaxy evolution experiment. We now consider a total processed FoV which scales as:

$$\Omega_F(\nu) = A(\nu/800)^{-5} \qquad \nu < 800\ \mathrm{MHz}$$

Again matching data rates gives A ~ 100 sq degrees at 800 MHz. If we select an integration time to detect an M* galaxy at z=1.5 and use this scaling we obtain a survey speed as a function of redshift which enables us to detect all M* galaxies out to z=3 over 1400 sq degrees in 390 days of integration.

A further tradeoff is the maximum baseline out to which we require the AA. This is not necessarily the limiting baseline for imaging at the frequencies covered by the AA, if the dishes are able to operate at the AA frequencies; instead this maximum baseline is determined by the need to avoid confusion in survey modes. The maximum baseline required for the HI experiments is relatively modest. For the dark energy experiment the maximum baseline is determined by the smallest-scale structure at a given redshift we wish to probe and this is usually assumed to be 5km. For experiments which require measuring the HI content of individual galaxies we can use the results of Abdalla and Rawlings who predicted the differential source counts in redshift bins – this is appropriate for this calculation since we can use position-velocity space to do the confusion analysis. The source count is of order 4×10^4 per sq degree at z=1 and is relatively flat to lower z. Using the normal criteria for source separation to avoid confusion (one source per 30 beam-areas on average), this translates to a required resolution at 700MHz of 3 arcsec or a baseline of ~ 35km similar to that given in SKA memo 83.

The continuum surveys require much longer baselines to avoid confusion. For the parameters of the all-sky HI survey above, assuming we take the available 300MHz bandwidth where the AA is fully sampled and process as a continuum data then we get the following sensitivity:

$\sigma(800) = 68$ nJy	equivalent to	42 nJy at L-band
$S_{\mathrm{lim}}(800) = 340$ nJy	equivalent to	210 nJy at L-band

Taking the Windhorst counts and extrapolating them we get a pessimistic requirement of 400 mas resolution or a 200km baseline. Furthermore since the imaging needs are relatively modest to avoid confusion it may not be necessary for all stations out to even this baseline to have an AA. An important question is: can we use AA out to baselines long enough to avoid confusion in the wide-field surveys (200km) and correlate against dishes on longer baselines for high sensitivity deep high-resolution surveys?

4. Specifications

The analysis of the previous section has shown that an AA can deliver excellent survey speed especially for HI experiments out to high redshift. Can this design be achieved? The cost is dominated by the number of elements and this in turn is dominated by the upper frequency at which the AA is fully-sampled. Taking this frequency to be 800MHz gives a bandwidth of 300MHz over which the AA is fully sampled – reducing this frequency much below 800MHz would reduce the available bandwidth (and hence continuum sensitivity) over which high dynamic-range observing is possible. More importantly 800 MHz already corresponds to a redshift of 0.75 for HI and reducing this makes doing the dark energy experiment very difficult.

The system temperature at 800MHz is taken to be 36 K consisting of a receiver temperature of 30K and a sky contribution of 6 K. By fixing the antenna separation appropriate for $\lambda/2$ spacing at 800MHz, a sensitivity within 5km can be obtained of 6000 m^2/. The main low-frequency survey experiments can then be achieved with a survey speed of 2×10^{10} m^4 K^{-2} deg^2. This gives a mean number of beams across the band of about 1200, a station diameter ~85m and an instantaneous observed Field of View of 200 deg^2 at 700 MHz. 66% of the collecting area is within 5 km. The remaining collecting area needs to be distributed so as to avoid confusion in continuum surveys which gives an upper baseline length for the AA of ~200km. To provide higher-resolution imaging it will be necessary to cross-correlate AA and dishes over the frequency range in common between the technologies.

The cost of such a system will be examined in detail in a forthcoming update of the SKADS design and costing work – we estimate that the full AA can be deployed for a cost of ~€400m plus infrastructure and central facilities to be shared between the AA and dishes.

The analysis discussed here was used as input to the specification Tiger Team review and the resulting full specifications are given in that document.

In conclusion we consider that a dense aperture array operating at significantly higher frequencies than has previously been used in radio astronomy can deliver the performance required for the demanding surveys in the SKA science case. The detailed frequency range, configuration and implementation as a system with dishes require careful future simulation and analysis in collaboration with the SKA Programme Development Office.

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