



Memo 65

SKA Signal processing costs

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Introduction

The delay in the building of the SKA has a significant impact on the signal processing cost of the SKA as it allows more time for Moore's law to reduce cost. In this paper the maximum SKA survey specifications are derived and it is found that only a few concepts are excluded on the basis of signal processing cost. It also proposes the use of regular rectangular arrays for antenna stations. This further reduces cost. For beamforming, a row-column approach to beamforming can be used. This reduces beamforming cost by about the square root of the number of antennas in the antenna station. For correlation the regular array forces all the sidelobe energy into grating lobes. By imaging the grating lobes correlator costs can be reduced by up to a factor of ten.

With row-column beamforming and imaging of grating lobe the maximum cost of signal processing is tens of millions of dollars. The implication for the SKA is that the sensitivity is almost totally dependent on the cost of the reflector and feed structure. For reflector based design this implies a reflector diameter or width of about 5m to minimise cost and ensure operation down to 300 MHz.

SKA specification

From the SKA web site the current SKA specifications that are relevant to the cost of signal processing in a wide field of view design are:

Instantaneous bandwidth of each observing band	Full width = 25% of observing band center frequency, up to a maximum of 4 GHz BW for all frequencies above 16 GHz
Simultaneous independent observing bands	2 pairs (2 polarizations at each of two independent frequencies, with same FoV centers)
Contiguous imaging field of view (FoV)	1 square degree within half power points at 1.4 GHz, scaling as λ^2 , 200 sq. deg. within half power points at 0.7 GHz, scaling as λ^2 between 0.5-1.0 GHz
Sensitivity at 45 degrees elevation (A/T)	Goal: 2500 at 60 MHz 5000 at 200 MHz, 20000 between 0.5 and 5 GHz, 15000 at 15 GHz, and 10000 at 25 GHz Goal: 5000 at 35 GHz
Survey speed	$FoV \times (A/T)^2 \times BW = 3 \times 10^{17} \text{ deg}^2 \text{ m}^4 \text{ K}^{-2} \text{ Hz}$ at 1.5 GHz $FoV \times (A/T)^2 \times BW = 1.5 \times 10^{19} \text{ deg}^2 \text{ m}^4 \text{ K}^{-2} \text{ Hz}$ at 0.7 GHz

With two simultaneous observing bands of 25% bandwidth the bands could be chosen at 622MHz with a bandwidth of 156MHz and 800 MHz with a bandwidth of 200 MHz. Thus the two simultaneous bands would cover a total bandwidth of 356 MHz. The Sensitivity specification at 200 MHz is equivalent to a total effective aperture of one square kilometre assuming T_{sky} of 170K and 30K contribution from the receiver,

spillover etc. The same effective area is needed at 0.5 GHz assuming T_{sky} is 20K. Assuming an aperture efficiency of 0.7 then the total collecting area that would be built is 1.43 square kilometres.

The specified field of view at 0.7 GHz is 200 square degrees and taking the product of the bandwidth, field of view, and the square of the sensitivity gives $2.84 \times 10^{19} \text{ deg}^2 \text{ m}^4 \text{ K}^{-2} \text{ Hz}$. This exceeds the specified survey speed indicating one of the parameters can be relaxed. But, by using what are effectively maximum parameters for bandwidth, field of view, and the square of the sensitivity a bound is set on the cost of the various signal processing components. So, for the current discussion it will be assumed that the bandwidth, field of view, and the sensitivity are as specified. For specification that differ from these the costs will scale directly with the change in the parameters except for the correlator where the cost scales as the square of the sensitivity

Signal Transport

To minimise the data transport in a wide field of view SKA it is proposed that beam data from individual antennas be transported to the central site [Bunton 2005]. In this case the result given in [Bunton 2005] shows that the total number of beams that must be transported to the central site N_b is

$$N_b = (\text{FoV}/\lambda^2)(S_{ska} T_{sys}/\kappa_a) \quad (1)$$

The 200 square degree field of view in steradian divided by the square of the wavelength at 0.7GHz is equal to 0.33 steradians/m² and the product of sensitivity S_{ska} times T_{sys} is equal to the effective area: 1,000,000 square metres. With κ_a equal to 0.7 [Bunton 2005] the total number of beams N_b is equal to 474,000.

If beamforming is done at the correlator then the data being transported to the correlator should be of a higher precision than the correlator precision. If not there will be reduced correlator efficiency due to the added noise as well as increased difficulty in calibrating the correlator. Most current designs assume a 4+4 bit complex input to the correlator. So for this paper it is assumed that the data transport uses 6+6bit complex data at cost of 12 bits/s per Hertz. With a total bandwidth of 356 MHz the data rate to the central site is 2×10^{15} bits/s if all antennas are used for wide field of view imaging. On long baselines it will be impossible to image the full field of view and here it is assumed that full field of view imaging will be limited to baselines of 150 km or less. For 150 km baselines a full 200 square degree image will contain more than a billion pixels at 0.7 GHz. Even this may be difficult to achieve. The SKA specification calls for 75% of the collecting area to be within 150 km of the central core, which reduces the wide field of view data rate to 1.5×10^{15} bits/s or 2000 Tbit/s. The 80% of this data, if not all, will use low cost fibre systems, such as VCSELs. In the 2002 white paper in cylindrical reflectors the estimated cost of such fibre systems in 2010 was US\$6,300 per Tbit/s. For the other 20% of data the cost will be about ten times higher. With this crude estimate the wide field of view signal transport cost is US\$15 million. As the SKA is to be built some five years later, in 2015, this is probably a conservative estimate but reducing the cost will not significantly affect the total SKA cost.

Filterbanks

The model proposed for beamforming at an antenna is first pass the data through a filterbank and then use simple weight and sum beamforming [Horiuchi et al 2004, Hall 2002, Bunton 2005]. If there is no antenna beamforming then the filterbank is still needed for the FX correlator. Thus every signal that comes from an antenna must be processed by a filterbank and the total number of signals that come from the antennas is equal to the total number of beams generated multiplied by a concept dependent constant, R_{fb} [Bunton 2005]. The constant R_{fb} is the ratio of the number of inputs to an antenna beamformer divided by the number of independent output beams and varies from 1 for a dish with a single feed to 4 for an aperture array with RF beamforming. In some concepts the R_{fb} is frequency dependent. For example for an focal plane array on a parabolic dish it is estimated to be 2.4 at the maximum operating frequency and increases to 9.6 at half this frequency. This occurs because the number of beams is approximately proportional to the square of the frequency. For the moment discussion will be restricted to operation of a feed array at its maximum frequency.

The minimum value of R_{fb} is 1. Using this value will give a minimum cost for filterbanks. For each real data sample into the filterbank approximately 20 real multiplies are needed. The total data rate into the filter banks is 474,000 beams times 356 MHz times 2 samples/s/Hz. This give an input data rate of 337 Tsamples/s and a total multiply rate of 6750 Tmultiplies/s. The current cost of multipliers, using a Virtex4 LS35 as a reference, is about US\$4000 per Tmultiplies/s. If the SKA is to be built in 10 years time and if Moore's law were to operate until the building of the SKA then the cost of multipliers would be reduced by a factor of 90. Even using a more conservative reduction by a factor of 32 brings the cost down to US\$125 Tmultiplies/s. As a rough estimate, total system cost is double the FPGA cost bringing the system cost to US\$250 Tmultiplies/s and the total cost of filterbanks is US\$1.7 million. For concepts where R_{fb} is greater than 1 this becomes US\$1.7 million $\times R_{fb}$.

Correlator

The cost of filterbanks and signal transport are low and vary little from concept to concept, but this is not true for the correlator. As a first step the maximum cost of the correlator is estimated. This occurs when each beam has the maximum field of view or 200 square degrees and there are 474,000 signals into the correlator which must be correlated together. An example of such a design is 474,000 2-m dishes. If the central 75% of these dishes are used for wide field imaging then there are 250 billion correlations (full Stokes parameters). Each correlation is a complex multiply of 4+4-bit data and in an FPGA implementation this is one quarter the cost of a multiply in the filterbanks and there are four multiplies per correlation. This gives a correlation cost of US\$250/THz. Each correlation has a bandwidth of 356 MHz and costs US\$0.089 each giving a total cost of US\$22.5 billion. For larger diameter dishes the cost per beam scales as the inverse of area squared and the number of beams is proportional to the area. So for a 6m dish the correlator cost is US\$2.5 billion, for a 15m dish it is US\$400 million, and for a small cylindrical reflector with an area of 600m² the cost is US\$118 million.

This greatly exceeds the available budget and a realistic target for the correlator cost is US\$100 million or less. A method of reducing the cost is to increase the size of the antenna. Increasing the antenna area by a factor N means each antenna must generate N times the number of beams but for each of these beams the number of correlations is reduced by N^2 . Thus the net reduction in correlator computations is a factor of N . To achieve a US\$100 million cost the antenna area would increase to 700 square metres or a 30-m diameter for a parabolic dish. However, correlator cost is directly proportional to survey speed. So reducing the target survey speed by a factor of four to $0.7 \times 10^{19} \text{ deg}^2 \text{ m}^4 \text{ K}^{-2} \text{ Hz}$ would allow 15-m diameter dishes to be used. Alternatively, large aperture antennas such as cylindrical reflectors or aperture arrays could be used.

Another method of increasing the antenna area associated with a beam going to the correlator is to form antenna stations out of groups of antennas. With arrays of parabolic dishes the beam from an antenna station has about one tenth the area of a beam generated by a single dish of the same total area. This assumes the dishes are separated by about three times their diameter to allow low elevation observation without blockage. The decrease in beam area has the effect of increasing the correlator cost by a factor of ten. To counteract this, the total collecting area of the antenna station must be greater than 7,000 square metres independent of the size of the dish. For an array of cylindrical reflector the beam area decreases by about a factor of three and an antenna station would need a total collecting area of greater than 2,000 square metres.

A further reduction in correlator cost or antenna station area might be possible if antennas are arranged on a rectangular grid and the grating lobes imaged [Bunton and Hay 2005]. By using a regular array of antennas in an antenna station, the station sidelobes degenerate to grating lobes with each grating lobe having the same array gain as the main lobe. Each grating lobe has the same gain and area as a beam purposefully formed in the direction of the grating lobe, so imaging in the grating lobes has the same sensitivity as imaging in the primary beam. For an array of parabolic dishes spaced at three times their diameter there are ~ 9 grating lobes and imaging of the grating lobes increases the area imaged by a factor of ~ 10 . Imaging of the grating lobes in arrays of cylindrical reflectors increases the area imaged by a factor of ~ 3 . Thus imaging of grating lobes would allow the minimum area of an antenna station to be reduced to 700 square metres independent of the type of antenna. However, imaging of grating lobes is only possible on short baselines where the grating lobes of different antenna stations coincide. Imaging of grating lobes is also an unproven concept. It is a promising concept that needs further work to prove its viability.

In summary the correlator for a wide field of view SKA will cost \sim US\$100 million if

- (a) each antenna has collecting area of $\sim 700 \text{ m}^2$, as is possible with aperture arrays, LAR, KARST and cylindrical reflectors
- (b) antenna stations made from a number of parabolic dishes have a total collecting area of $\sim 7000 \text{ m}^2$.
- (c) antenna stations made from a number of cylindrical reflectors have a total collecting area of $\sim 2000 \text{ m}^2$.
- (d) imaging of grating lobes is possible at large enough baselines and the antenna station is a regular array of antennas with a total collecting area $\sim 700 \text{ m}^2$.

The cost of the correlator is reduced if the collecting area of the station or antenna in any of the above options is increased. For example, if single antennas have a collecting area of $\sim 1400 \text{ m}^2$ then the correlator is estimated to cost US\$50 million. Correlator cost is also directly proportional to survey speed with the above estimates assuming a survey speed of $2.8 \times 10^{19} \text{ deg}^2 \text{ m}^4 \text{ K}^{-2} \text{ Hz}$. Reducing the survey speed to $1.5 \times 10^{19} \text{ deg}^2 \text{ m}^4 \text{ K}^{-2} \text{ Hz}$ reduces the cost to US\$53 million or reduces the required collecting areas to 53% of the values given above.

Beamforming

For most beamforming applications the collecting area is proportional to the number of elements beamformed, for example in a phased array. In this case the beamformer and correlator cost are inter-related. For parabolic dishes with focal plane arrays this is not the case, the effective area of any beam from the antenna is approximately constant. As the latter case is simple it will be dealt with first.

Current estimates at CSIRO show that at the highest frequency of operation reasonable aperture efficiency and T_{sys} are achieved if 20 dual polarisation signals are used to form each beam. Thus each beam is the complex sum of 40 signals and the weighting required for each signal is a complex multiply or four real multiplies. Thus 160 real multiplies are needed per beam. With a bandwidth of 356 MHz this is estimated to cost US\$14 per beam. For the 474,000 dual polarisation beams of the wide-field SKA this comes to US\$13 million making it a comparatively insignificant cost. It also indicates that the actual number of elements that must be used in forming a beam from a focal plane array on a parabolic dish is not an important factor in terms of SKA costs. However a limitation of this design is that the field of view does not increase with wavelength. If the 200 square degree field of view occurs at the lowest frequency of operation of an octave wide focal plane array then the cost increases by a factor of four.

For beamforming where the area is proportional to the number of elements it will be assumed that the maximum beamforming cost will be US\$100 million and there are three cases to consider: minimum station area equal to 700, 2000 and 7000 m^2 . The 700 m^2 case covers single antennas of this area and arrays of antennas of this area where grating lobes are imaged. For this case the number of dual polarisation beams that must be formed is 474,000 and to achieve the cost objective each must cost less than US\$105. The cost of adding each signal to the sum is $\$0.25/\text{Gmultiplies/s}$ multiplied by 4 multiplies/Hz multiplied by 0.356GHz or US\$0.089 per input. At this cost, up to 1180 inputs can be summed to generate each beam. As the total area of the station is at least 700 m^2 each input must have an area greater than 0.6 m^2 or about 0.85m across.

For arrays of parabolic dishes with focal plane arrays there are tens of antennas per antenna station. With a station collecting area of 7000 m^2 the correlator cost is reduced to US\$10 million and there are forty 15m antenna per station. The beamforming cost is \$7 per dual polarisation beam and US\$3.5 million for the full SKA. Reducing the antenna size to 6m increases the number of antennas per station to 250 giving a total beamformer cost of US\$20 million. For a 2m dish there are 2228 antennas and the cost of beamform for random antenna placement is US\$195 million.

It would appear that with 2m dishes the size of a station must be less than 7000 m² to reduce beamforming cost but correlator costs increase. To achieve acceptable correlator cost the antennas must be arranged in a regular array, allowing the beamforming to be done hierarchically in a row then column manner. If there are 2209 2m dishes in an antenna station then they could be arranged in a 47 by 47 grid and the beamforming done first on the rows and then on the columns. At each stage there are 474,000 dual polarisation beams and the cost of each intermediate dual-polarisation summation is US\$8 (sum of 47 signals). With a sum of rows then a sum or columns total cost is US\$8 million, while still maintaining a 7000 m² antenna station. For 6m antennas the use of row-column beamforming reduce the cost from US\$20 million to US\$2.7 million

When RF beamforming is used on a cylindrical reflector the field of view at 700 MHz of the signal coming from the antenna needs to be about 400 square degrees. This can be reduced by the use of parallel linefeeds or narrower reflector widths. But in the worst case the equivalent collecting area associated with each signal is half that of a 2m parabolic dish and there are about 4761 signals to beamform in a 7000 m² antenna station. Optimally this is processed by breaking the linefeed into 69 RF beamformed sections. Each beam from these sections cost US\$6 to process. There are now 69 inputs into the next stage beamformer and the total cost for 474,000 dual polarisation beams is \$12 million. Reducing the width of the reflector by a factor of four increases the number of feed elements by four but also increases the field of view by a factor of four. To maintain the field of view constant, RF beamforming is needed which keeps the signals to be processed constant and hence the beamforming cost is independent of reflector width.

If imaging of grating lobes is not possible then 4,740,000 dual polarisation beams must be generated in an SKA composed of parabolic dishes formed into antenna stations. The cost of generating each beam must be less than US\$10.5 and the number of inputs per beam is reduced to 118. As the correlator requires antenna stations with an area greater than 7000 m², the area associated with each input is more than 60 m². This shows that the minimum parabolic dish antenna diameter for this case is 8.7m. This would appear to rule out parabolic dishes with single feeds in a random configuration as a possible SKA solution. For a 15m parabolic dish with a focal plane, the beamforming cost is estimated to be US\$35 million. It seen that the correlator cost dominates but with 200 antenna stations each 7000 m² in area there is little scope for increasing the station size and hence reducing the correlator cost.

Cylindrical reflectors can be used individually. If the area per cylinder is 600m² then each of the 2500 cylinders must generate 190 beams. As ratio R_{fb} of the number feeds per beam equals two for a cylinder there are 380 inputs per beam. With a 356 MHz bandwidth this is 541T multiplies/beam or US\$135 per beam for a total cost of US\$64 million. When cylindrical reflectors are used in antenna stations and grating lobes are not imaged 1,422,000 beams are needed. The cost of generating each beam must be less than US\$70 and this cost is distributed over linefeed beamforming and beamforming between cylinders. As with the regular array the line feeds can be broken up into 69 elements that generate a signal going to the beamformer. Thus the first stage beamforming is the same cost as that of the regular array: US\$6 million. The second stage of beamforming generates three times as many beams but the cost

per beam is the same and will cost US\$18 million. The total beamformer cost for a 7000 m² antenna station is US\$24 million. For a regular array the cost is US\$12 million and is independent of reflector width.

For aperture arrays the antenna station must be broken up into ~1 square meter apertures for a total of 7000 inputs into the beamformer. These are then processed as 83 rows and 83 columns. The cost of a single beam is US\$14 for a total beamformer cost of US\$7 million.

Table 1 Beamformer and Correlator cost for a number of concepts with 7000 m² antenna station

	Correlator cost US\$ millions	Beamformer cost US\$ millions	Total cost US\$ millions
2 m dishes single feed			
Full correlator	22,000	small	22,000
random antenna distribution	100	195	295
regular antenna distribution	10	8	18
6 m dish with focal plane array			
Full correlator	2,5000	13	2,500
random antenna distribution	100	20 + 13	133
regular antenna distribution	10	2.7 + 13	25.7
15 m dish with focal plane array			
Full correlator	400	13	400
random antenna distribution	100	3.5 + 13	116.5
regular antenna distribution	10	1.7 + 13	24.7
Cylindrical reflector (600 m ²)			
Full correlator	118	64	182
Cylindrical reflector (any width)			
random antenna distribution	30	24	54
regular antenna distribution	10	12	22
Aperture Arrays	10	7	17

Cost Scaling

The cost in Table 1 is given for cases where the antenna station size is 7000m² and the survey speed of $2.8 \times 10^{19} \text{ deg}^2 \text{ m}^4 \text{ K}^{-2} \text{ Hz}$. It is likely that survey speed will be less than this and for imaging fidelity the station size might also be smaller. In all cases the cost is proportional to both field of view and bandwidth. For example halving the field of view requirement halves the survey speed as well as all the costs in Table 1.

The scaling of cost with antenna station size and total area is more complex. There are two simple cases to consider: fixed A/Tsys and fixed station size. If the station size is fixed then a change in A/Tsys means a change in the number of antenna stations N_a . Thus the beamforming cost is proportional to A/Tsys and the correlator cost is proportional to A/Tsys squared. For example, halving the area halves the beamforming cost and reduces the correlator cost and the survey speed by a factor of four. This would make a full correlator for 15 m dishes viable but is the reduction in sensitivity and survey speed acceptable?

If A/T_{sys} is fixed then the number of antenna stations N_a could be varied. The proposal here has 200 antenna stations which is close to the minimum for good UV coverage and image quality hence a larger N_a should be considered. For stations with random antenna spacing the number of inputs per beam and hence total beamformer cost is proportional to N_a^{-1} . For regular arrays of antennas the cost is proportional to $N_a^{-1/2}$. In all cases the cost of the correlator is proportional to N_a as the number of correlations per beam increases as N_a^2 but the number of beams per station is proportional to N_a^{-1} . Thus increasing N_a does little to change the cost of 6m dishes with random arrays as the decrease in beamforming cost is balanced by the increase in correlator cost. But for many of the possible concepts and configurations the correlator cost is low enough that an increase in N_a is possible.

Implications for the SKA

It is seen that the signal processing cost is probably insignificant if row-column beamforming with regular antenna arrays is used. Correlator costs are also low if imaging of the grating lobes is used. In this case the cost of building the SKA will be largely dependent on the cost the antenna elements. The cost in Table 1 are for roughly double the SKA survey speed specification so just meeting the specification would halve the cost making random arrays with dishes as small as 6m viable as well as all cylinder and aperture array configurations. Thus it is seen that all concepts are viable except for small dishes with a full correlator and 2m dishes with random arrays.

For reflector based designs the cost of a reflector increases with increasing diameter or width. Thus minimum reflector cost corresponds to a minimum size. Assuming a LOFAR/Mileura Widefield Array system is used at frequencies up to 300 MHz and the reflector has reasonable aperture efficiency when their size is five wavelengths across then the minimum cost antenna has dimensions of 5m. A single feed dish design would give a 27 square degree field of view at 0.7 GHz, which may be sufficient especially if greater area is built to compensate for the smaller than specified field of view. Adding a small focal plane array to the dish will allow field of view requirements to be met. For a 5m cylindrical reflector RF beamforming across two antenna elements is allowed with a 0.7GHz focal plane.

Conclusion

For a survey speed of $1.5 \times 10^{19} \text{ deg}^2 \text{ m}^4 \text{ K}^{-2} \text{ Hz}$ all concepts have acceptable signal processing costs except small dishes with a full correlator or in antenna arrays where the distribution of antennas is not regular. If the antenna arrangement is regular then correlator cost can be reduced by imaging grating lobes and the beamforming cost reduced by row-column beamforming. With these two innovations the correlator and beamformer cost for all antenna designs, including small dishes with a single feed, is reduced to about US\$20 million for a survey speed of $2.8 \times 10^{19} \text{ deg}^2 \text{ m}^4 \text{ K}^{-2} \text{ Hz}$.

The small cost of signal processing implies that SKA sensitivity is maximised by using the antenna with the lowest cost per square meter. For reflector antenna the cost is reduced by reducing the antenna size. If a lower operating frequency of 300 MHz is needed then the antenna size should be about 5m.

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