



# **Memo 71**

## **Focal-Plane Array Architectures: Horn Clusters vs. Phased-Array Techniques**

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SKA Memo 71

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## Abstract

The science goals of the SKA specify a field of view which is far greater than what current radio telescopes provide. Two possible feed architectures for reflector antennas are clusters of horns or phased-array feeds. This memo compares these two alternatives and finds that the beams produced by horn clusters fall short of fully sampling the sky and require interleaved pointings, whereas phased-array feeds can provide complete sampling with a single pointing. Thus for a given focal-plane area horn clusters incur an equivalent system temperature penalty of  $\sim 2\times$  or more. The situation is worse for wide-band feeds since the spacing of the beams is constant while the beamwidth is inversely proportional to frequency, increasing the number of pointings for a fully-sampled map at the high-end of an operating band. These disadvantages, along with adaptive beamforming capabilities, provide a strong argument for the development of phased-array technology for wide-field and wide-band feeds.

## 1 Introduction

Current science goals of the SKA require a telescope with a very large field of view below 1 GHz [1]. Fields of view on the order of 40 square degrees at 700 MHz would enable revolutionary dark-energy surveys to be completed on the timescale of a year [2]. A dark energy survey would require about half the sky to be mapped between 0.56 and 1.4 GHz. This specification presents a problem to engineers: how to design telescope to have a greatly expanded the field of view? One solution is to place a multi-pixel array of antennas and receivers at the focal plane of a reflector antenna, effectively turning the telescope into a radio camera. Two technologies exist to do this. The first is to place a cluster of conventional horns at the focal plane with each receiver output corresponding to a beam on the sky. The second method samples the

fields near the focal plane with an array of densely-packed wide-band antennas (such as Vivaldi antennas [3, 4]) and adds the outputs of several receivers together with complex weights to synthesize beams on the sky. The advantage of the horn cluster is that it is well understood and has been optimized to provide excellent performance on telescopes. In comparison, focal-plane arrays based on phased-array techniques are still under development, but many engineers believe that this technology is the best choice to construct very sensitive radio telescopes with very-wide fields of view. This memo will examine this choice of architecture by comparing it with high-performance horn clusters including geostationary communications satellites, the Parkes Multi-Beam Receiver, the FCRAO 15-beam millimetre receiver, and arrays that could be constructed using wide-band prime-focus feeds.

## 2 Representative Horn Clusters

### 2.1 Satellite Horn Clusters

Before examining the two reference astronomy receivers, another cluster feed system will be explored, that used by satellite operators to define the coverage area of geostationary communications satellites. Often coverage area is defined by imaging the earth onto the focal plane where it is sampled by an array of horns. Typically the spot beams produced by this cluster of horns overlap at the -6 dB level. This beam spacing is greater than that required for imaging and the illumination of the coverage area will not be uniform. Therefore ground stations must be designed to accommodate this spatial variation in signal level. In addition, there is another loss of efficiency from grating lobes (on the order of 1 to 1.5 dB) due to the horn spacing being greater than  $\lambda$  [5]. These deficiencies arise from the contradictory requirements to make the feed horns large enough so that the reflector is efficiently illuminated while having an inter-element spacing small enough to provide continuous coverage on the earth. In contrast, phased-array feeds have element spacings  $\leq \lambda/2$  (to prevent grating lobes) and the illumination properties are controlled by the beamforming network rather than the element size. Thus phased arrays can fully sample the focal fields, and equivalently, the sky.

### 2.2 Parkes Multi-Beam Feed

The 21-cm Multibeam Receiver on the Parkes Radio Telescope [6] is an example of a sensitive and efficient horn array (see Fig. 1). Thus it provides a good reference by which proposed wide-field arrays can be compared. To keep the array as densely-packed as possible, the horns use a stepped circular waveguide design without dielectric loading to minimize loss. A corrugated horn design is not used as it would increase the spacing by thickening the horn walls. In this array the horn spacing is  $1.2 \lambda$  and the beam spacing is 2 half-power beamwidths (HPBW). System parameters are summarized in Table 1. Note that the system bandwidth is only 16%.



Figure 1: The horn array of the Parkes Multibeam Receiver.

Table 1: Summary of Parkes Multibeam Receiver Parameters

Frequency Range	1.27–1.47 GHz
Number of Beams	13
Number of Polarizations	2/beam
Array Diameter	1.3 m
Array Length	1.35 m
Horn Diameter	24 cm
Beam Spacing	2 HPBW
$T_{rx}$	3K
$T_{sys}$	25K

Table 2: FCRAO 15-Beam Receiver Parameters

Frequency Range	90–115 GHz
Number of Beams	3×5
Number of Polarizations	1/beam
Horizontal Beam Spacing	1 HPBW
Vertical Beam Spacing	2 HPBW

### 2.3 FCRAO Multi-Beam Millimetre Receiver

The Five-Colleges Radio Astronomy Observatory constructed a 15-beam millimetre-wavelength receiver [7] based on corrugated feed horns. Although a 3×5 array of beams is produced on the sky, it is actually a 2×3 vertically-polarized array and a 3×3 horizontally-polarized array interleaved on the sky with a polarizing beamsplitter. For each subarray the beam spacing is two beamwidths, but with polarization interleaving one beamwidth spacing is obtained in horizontal rows. The array is fabricated with corrugated horns. To avoid increasing the horn spacing by twice the corrugation depth, the horn design is modified so that as the flare expands towards the aperture, the corrugations are truncated and replaced with smooth walls and the aperture profile changes from circular to square. The bandwidth of this array is 28%.

### 2.4 Wide-Band Prime-Focus Feeds

Unfortunately the examples presented above provide small fractional bandwidths. Future instruments will require a wide instantaneous bandwidth of an octave or more. What options are available to designers? Wide-band corrugated feeds have been fabricated for Cassegrain reflector antennas (for example the EVLA [8]), but a Cassegrain optical system is not suitable for wide-field imaging due to vignetting. Thomas, Greene, and James point out that it is difficult to design a horn that has both the wide beamwidth required for a prime-focus feed-point and a large bandwidth [9]. Thus very few designs exist in antenna literature. Thomas’ group present an unusual curved-profile corrugated design that achieves nearly an octave of bandwidth (see Fig. 2). Unfortunately there is significant variation of the beamwidth (~40%) over the band. Another design has been presented by Ying, Kishk, and Kildal which has a similar bandwidth but with much less beamwidth variation (~18%) [10].

Another important consideration is the diameter of the feed as this will determine the packing density. For the Thomas feed this is  $1.86\lambda$  and for the Ying feed it is  $2.1\lambda$ , both at the lowest frequency of operation. In comparison, the Parkes array has a spacing of  $1.2\lambda$ . Thus an array constructed with Thomas’ feed will have a separation between beams of 3.1 beamwidths, and an array using Ying’s feed will have a 3.5 beamwidth spacing.



Figure 2: A wide-band corrugated feed similar to that described by Thomas *et al.* [9]. Notice that the horn flare is curved rather than linear as is the usual case with corrugated horns.

Recently, several log-periodic antenna designs have been developed that go beyond octave bandwidth and have decade or more instantaneous bandwidth [11, 12]. However, it is not known how closely these antennas (or wide-band horn feeds for that matter) can be packed. Mutual coupling might degrade antenna performance. Thus wide-band feeds also require significant research and development before they are ready for deployment. However, for the following discussion it will be assumed that a packing density similar to that of narrow-band horns can be achieved.

### 3 Comparison of Horn Clusters and Phased-Array Feeds

#### 3.1 Field of View

The scientific need for very-large fields of view is pushing engineers to explore and develop multi-beam focal-plane arrays. The advantage of using densely-packed arrays with a beamforming network is that the focal plane is completely sampled (out to the extent of the array) and thus there is complete coverage within the field of view. In contrast, horn arrays sample separate focal spots, the distance between which is determined by how close the horns can be packed. Unfortunately the diameter of a horn is typically greater than the half-power diameter of the focal spot, resulting in beams on the sky that are spaced further apart than Nyquist sampling

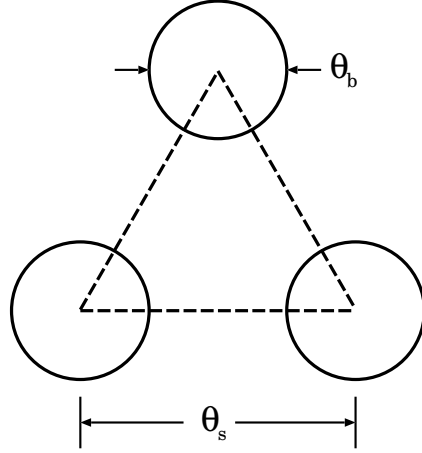


Figure 3: Three beams from a hexagonal cluster of feed horns. From this triangular unit cell the sampled beam area in comparison to the total area can be calculated.

theory requires. In comparison, phased-array technology allows the outputs of one array of antennas and receivers to be processed in any number of beamformers to provide additional beams with arbitrary spacing with respect to other beams on the sky. (For discussions on sampling theory as it relates to focal-plane arrays, see [13, 14].)

Consider the case with hexagonal spacing of feed horns. Figure 3 shows three beams from an array with beamwidth  $\theta_b$  and beam spacing of  $\theta_s$  forming a triangular unit cell. The area of the cell,  $A_c$ , is

$$A_c = \frac{1}{2}\theta_s \left( \frac{\sqrt{3}}{2}\theta_s \right) = \frac{\sqrt{3}}{2}\theta_s^2 \quad (1)$$

and the area of the beams within the cell,  $A_b$ , is

$$A_b = \frac{1}{2} \left( \theta_b^2 \frac{\pi}{4} \right) = \frac{\pi}{8}\theta_b^2. \quad (2)$$

The beam efficiency,  $\epsilon_b$ , is the ratio of the two areas,

$$\epsilon_b = \frac{A_b}{A_c} = \frac{\pi}{2\sqrt{3}} \left( \frac{\theta_b}{\theta_s} \right)^2. \quad (3)$$

For the Parkes Multibeam Feed the ratio of the beamwidth to the beam spacing is  $\sim 1/2$  so the efficiency is 0.23. Thus slightly more than 4 pointings are required to fully sample a region of the sky. An array consisting of Thomas horns would have  $\epsilon_b = 0.094$ , requiring  $\sim 11$  pointings and an array of Ying horns would have  $\epsilon_b = 0.074$  and require  $\sim 14$  pointings.

The situation is worse for square arrays such as the FCRAO system since

$$\epsilon_b = \frac{\pi}{4} \left( \frac{\theta_b}{\theta_s} \right)^2. \quad (4)$$

With 2×half-power beamwidth beam spacing the efficiency is 0.2.

### 3.2 Receiver Sensitivity Comparison

Using the results from the previous section the sensitivity specification for a phased-array feed can be derived. For a given focal-plane area a horn cluster is inherently slower because any single pointing only partially samples the sky, while a phased-array feed can fully sample a region of the sky with a single pointing. Thus a phase-array feed can have a higher system temperature. This is significant since it may be difficult for individual receivers in an ambient-temperature Vivaldi array to match the performance of cooled receivers in a high-performance horn array. However, this should not be a license to be sloppy with receiver design since it is important that phased-array feeds be significantly faster than horn clusters.

From the radiometer equation, the sensitivity of a telescope is

$$\Delta T \propto \frac{T_{sys}}{\sqrt{B\tau}} \quad (5)$$

where  $T_{sys}$  is the telescope system temperature,  $B$  is the receiver bandwidth, and  $\tau$  is the duration of the observation. The larger the field of view, then the longer the telescope can integrate at a particular pointing. So  $\tau \propto \Omega$  where  $\Omega$  is the solid angle of the field of view. The field of view can be related (neglecting the beam deviation factor) to the area of the focal plane ( $A_f$ ) using  $\Omega \sim (r/f)^2 \sim A_f/f^2$  where  $f$  is the focal length of the telescope and  $r$  is the radius of the focal-plane array. Thus the sensitivity equation (5) becomes

$$\Delta T \propto \frac{f T_{sys}}{\sqrt{B A_f}}. \quad (6)$$

Now suppose that there are two telescopes, one with a horn cluster with a system temperature of  $T_h$  and another with a phased-array feed with a system temperature of  $T_p$ . The telescope with a phased-array feed can instantaneously image a *filled* field of view  $\Omega_p$  so it has a beam efficiency of unity while the horn array has beams within a *sparsely*-filled field of view  $\Omega_h$  and a beam efficiency  $\epsilon_b < 1$ . For the two telescopes with the same bandwidth  $B$  to have the same sensitivity,

$$\frac{f_h T_h}{\sqrt{\epsilon_b A_h}} = \frac{f_p T_p}{\sqrt{A_p}} \quad (7)$$

or

$$T_p = \frac{f_h}{f_p} \frac{T_h}{\sqrt{\epsilon_b A_h/A_p}}. \quad (8)$$

Using the Parkes Multibeam Feed as an example, it was found in Section 3.1 that  $\epsilon_b = 0.23$ . Assuming that the phased-array feed and the horn cluster occupy the same area of the focal plane ( $A_p = A_h$ ) on the same telescope ( $f_p = f_h$ ), then  $T_p = 2.1 \times T_h$ . Thus with  $T_h = 25\text{K}$ , the phased-array feed will be competitive with

Table 3: Summary of Horn Cluster Performance

Horn Type	Element Diameter	Beam Spacing	Beam Efficiency	$T_p$ ( $T_h = 25\text{K}$ )
Parkes	$1.2\lambda$	2 HPBW	0.23	52K
Thomas	$1.86\lambda$	3.1 HPBW	0.094	82K
Ying	$2.1\lambda$	3.5 HPBW	0.074	94K

a system temperature less than 52K. The other feeds described in Section 2.4 are even worse because they sample the focal plane so sparsely. For an array of Thomas feeds, it will be slower than a phased-array feed with  $T_p \leq 82\text{K}$  and for Ying feeds  $T_p \leq 94\text{K}$ .

Table 3 summarizes the performance of horn arrays constructed with various types of feeds. Note that only the first array exists; the other two sets of results are based on the performance of individual feeds. The last column in the table lists the maximum noise temperature for a competitive phased-array feed assuming a horn element system temperature of 25K.

### 3.3 Bandwidth

Future telescopes will have a very large instantaneous bandwidth. Vivaldi array feeds are inherently wide band and Section 2.4 has shown that there are prime-focus horn feeds that have nearly an octave bandwidth. How well suited are these candidates for wide-field, wide-bandwidth surveys?

With feeds using phased-array techniques, as long as the frequency is below where the spacing is  $\lambda/2$  the focal plane is fully sampled and the field of view is frequency independent. In comparison, horn clusters cannot completely sample the focal plane. They illuminate the sky with a grid of beams, the centres of which are independent of frequency while the width of the beams is proportional to  $1/\nu$ . Therefore over an octave the beam efficiency is reduced by a factor of 4 (see (3)) and the sensitivity by a factor of 2 (see (8)).

How significant is this loss of sensitivity with frequency? The answer depends upon the type of observation made. For example, consider a dark-energy survey. At high frequencies the nearby universe is probed where the volume is smaller, sources are closer, and thus less sensitivity is needed. It is at lower frequencies where the dark energy signature is found and full sensitivity is needed as the signals are weaker and a much larger volume of space is mapped. Thus variable field of view observations are *not* a handicap for this particular experiment [2], but may be for others.

Table 4: Scanning Angle for 1 dB Loss vs. Focal Ratio

$f/D$	Offset [HPBW]
0.4	3.5
0.5	5
0.6	7
0.7	10
0.8	13
1.0	19.5

### 3.4 Other Problems with Horn Arrays

There are two other problems with horn arrays. The first is that if mounted on a telescope with an altitude-azimuth mount the field of view will rotate as the telescope tracks. Several solutions exist. One is to mount the array on a positioner so that it can be rotated to follow the rotation of the field. This is unattractive as it adds another mechanical component to the system, something subject to wear and that requires maintenance. For very large surveys, another solution is to take a large number of short snapshots at different pointing angles, and then to de-rotate and combine the snapshots. In this case “short” is defined as a period of time less than that for a source to be rotated out of an outer beam. The last solution is to use an equatorial mount. Field rotation is easily accommodated with a phased-array feed since the field is fully sampled and can be de-rotated by rearranging the output data.

The other problem with horn arrays is that horns far from the field centre have large optical aberrations, such as coma, resulting in reduced efficiency. This problem is not nearly as severe for focal-plane arrays using phase-array techniques since beamformer weights can be modified (eg. with conjugate field matching) for non-central beams to match distorted focal spots. Unfortunately, horns cannot be adapted to match non-central focal spots and the only method to reduce off-axis distortion is to increase the focal length of the reflector antenna. Milligan [15] (p231, Fig. 8-7) has a plot of scanning loss (phase-error loss) that illustrates this effect. These results are summarized in Table 4 which tabulates the maximum scanning offset for which the efficiency is reduced 1 dB (reduced to 0.8) as a function of  $f/D$ . This shows that large fields of view are possible but require a telescope with a focal length much longer than usual.

### 3.5 Advanced Capabilities of Phased-Array Technology

Phased-array technology provides several capabilities beyond providing a large number of densely-packed beams. Other features are possible because the focal-plane aperture illumination can be tailored to use the optical system in an optimal way.

**Off-axis aberrations:** The size of the field of view for horn arrays is limited by off-axis aberrations, such as coma. However, because a phased-array feed can fully sample the field distribution in the focal region, a conjugate match of the beamformer weights to a distorted focal spot results in an off-axis beam shape that is the same as that of the central beam [16].

**Optimal beamforming:** Traditional feed horns can do little to ameliorate the effects of scattering by feed legs or blockage by the prime-focus receiver. Adjusting beamformer weights can reduce the feed-array gain in the direction of these obstacles [17]. Optimal weights can be determined using eigenfilter techniques [18, 19, 20].

**Frequency-invariant properties:** Another problem with traditional feed horns is that the beam properties are a function of frequency. This means that the reflector is optimally illuminated at only one frequency. This deficiency can be avoided by phased-array feeds that use beamformer weights that are a function of frequency [21].

**Compensation of reflector-surface distortion:** Large-scale distortions in the reflector surface (due to gravitational loading or from low manufacturing accuracy) can be compensated by an array feed with a beamformer [22].

## 4 Discussion

The most significant advantage of a phased-array feed (such as one constructed with densely-packed Vivaldi elements) is that it is capable of fully sampling the focal plane of a radio telescope while a horn cluster cannot. Due to reduced beam efficiency, typically  $\sim 4$  or more pointings by a horn cluster are needed to fully sample a region of the sky. This efficiency is reduced even further with wide-band receivers since as the observing frequency is increased, the width of the horn-cluster beams are reduced. For example, doubling the observing frequency increases the observing time by another factor of 4. Together, these two factors form a significant disadvantage for horn arrays.

The beam-efficiency advantage of phased-array feeds will allow designers to use noisier receivers and still have a telescope with a faster survey speed than a telescope equipped with a horn cluster. This is important because the smaller number of receivers in a horn cluster allows more expensive technology (such as cryogenic cooling) to be used to achieve lower system temperatures, but this advantage can be offset by the mapping efficiency of phased-array feeds.

The last advantage of phased-array technology is the adaptive nature of the beamformer. This allows for a very flexible system which can be optimized and correct for non-ideal properties in telescope optics. If the beamformer is frequency-dependent, then it can be programmed to synthesize feed properties that are constant across the observing band.

The most significant advantage of horn clusters is that it is a mature, well-understood technology. However, this qualification only applies to narrow-band clusters since

*wide-band* horn arrays have yet to be deployed and thus cannot be considered mature technology. Further research is necessary to develop wide-band designs that can be packed more densely on the focal plane. It may also be desirable to build reflector antennas with larger  $f/D$  to reduce the aberrations at the edge of the focal-plane array. While low-noise phased-array feeds are probably even less mature, the advantages of phased-array technology are so significant that it cannot be ignored.

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