Memo 98

Definition of Array Receiver Gain and Noise Temperature

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Abstract

With the emergence of aperture and focal plane array systems for application in radio astronomy, there is a need for clear definitions of their gain and noise temperature, preferably in terms of a single antenna/receiver system, to enable proper understanding and comparison of system performance. Although in the literature the noise temperature of a phased array receiver was defined in these terms a long time ago (1966), this paper is not well known, either because it is not easily accessible, or slipped from our common memory. This memo reassesses the original paper with respect to the definitions and places them in the context of radio astronomical applications of arrays with strong coupling between the array elements. Essentially the array antenna and receiver are replaced by a single antenna and single receiver having the overall properties of the antenna array and individual array receivers, respectively (with arbitrary gains and noise temperatures of the individual receivers). An example will be given for the calculation of the gain and noise temperature of an array receiver, starting from the properties of the individual receiver channels in the array environment.

1 Introduction

When considering array systems with multiple antenna inputs and receiver channels, it is important to understand how the concept of system gain and input noise temperature, originally defined for two-port systems, should be interpreted for these array systems. The starting point is the definition of noise temperatures in a single-channel receiving system, consisting of an antenna, an LNA and subsequent receiver stages and a transmission line connecting the LNA to the antenna feeding structure. In general the antenna is considered as a radiator (receptor), which for radio astronomy applications may consist of a (parabolic) reflector antenna and a feeding structure. Section 2 defines the noise temperatures relevant for this system, as well as a reference point for the receiver and system input noise temperature. For a conventional reflector system with a nearly lossless feed, this reference point may be defined at the feed input or output without affecting the resulting system noise temperature. For aperture and focal plane arrays the reference point should be chosen at the array input, to enable the definition of the noise temperature as a measurable quantity. To be able to apply the receiver temperature definition for a single-channel system to an array system, section 3 shows how the array system will be converted to an equivalent system, consisting of a single-output antenna and a two-port receiver. Antenna specific properties like beam shape and direction, as well as aperture efficiency, will be modeled in a lossless single-output antenna. The multiple receiver channels, including the effects of beam forming and array properties, will be described by one two-port, characterized by an overall gain and noise temperature. The (system) noise temperatures defined for a single receiver system in this section may then be used for the equivalent array system. The basic idea for this approach is mentioned in [1], where a figure of merit G/T for active array antennas is defined, following a statement that for phased arrays G and T are not very well understood parameters by many system designers. This was already realized by Waldman and Wooley in 1966 ([2]), who made the original description of the gain and noise temperature of a phased array receiver. The latter is more fundamental in defining the array and receiver gain, as well as receiver and system noise temperatures. Therefore it will be used in this document to define and clarify the array properties and will serve as a reference for application in and understanding of phased array systems in general. The calculation of the equivalent gain and noise temperature of a phased array is shown in section 4. The results will apply to any phased array system, either finite or infinite, aperture or focal plane arrays. This is illustrated in section 5, where as an example the gain and noise temperature of the equivalent FARADAY FPA ([3]) two-port receiver are calculated step by step through the various levels of beam forming.

2 Definition of noise temperatures in a receiving system

The sensitivity of antenna receiving systems is generally expressed as the ratio of antenna gain or antenna effective area and system noise or antenna temperature (G_a / T and A_{eff} / T, respectively). For a single receiver system with an antenna, e.g. a reflector antenna with a receiver in the primary focus, the antenna gain or effective area, as well as the system temperature are defined at the antenna output terminals/receiver input (see Figure 1). In this case the antenna output terminals and receiver input are defined at the launcher input of the feed illuminating the antenna reflector surface. The system noise temperature comprises the receiver noise temperature, which includes the LNA and second stage noise (T_{LNA}, defined here including the second stage contribution), as well as noise due to losses between LNA and antenna/feed output. These losses are described with a power transmission coefficient $\alpha \leq 1$ at a physical temperature T_{amb}, for the overall receiver temperature leading to: $T_{rec} = (1/\alpha - 1)T_{amb} + T_{LNA} / \alpha$. Loss due to the feed or an antenna element may also be included in α, thus representing the situation of a lossless feed as discussed in the Introduction. Furthermore antenna related noise contributions like spill-over T_{sp} and side-lobe noise T_{sl}, as well as noise due to reflector mesh transmission T_{mt} add to T_{sys}: $T_{spill} = T_{sp} + T_{sl} + T_{mt}$. On top of that the antenna receives noise from celestial sources, i.e. sources of astronomical interest T_s, including sky noise T_{sky} and cosmic background noise T_{cmb}, sometimes referred to as antenna temperature T_a: $T_a = T_s + T_{sky} + T_{cmb}$. The value of this system noise contribution is modified by the antenna aperture efficiency (η_A), which also determines the effective gain (G_a) and thus the effective area (A_{eff}) of the antenna. For the overall system temperature and effective area at the reference point of the antenna feed, this results in: $T_{sys} = T_{rec} + T_{spill} + \eta_A T_a$ and $A_{eff} = A_{phys}\eta_A$, with A_{phys} the physical collecting area depending on the size of the antenna.
We use the notation T_{sys} for the effective antenna system temperature at the feed/receiver input and define T_a as the noise contribution from celestial sources. A_{eff} is used here instead of antenna gain G_a for two reasons: first A_{eff} is more relevant for the discussion about SKA sensitivity (defined as A_{eff} / T_{sys}) and it is more closely related to the physical properties of antennas envisaged for SKA than G_a; secondly it avoids confusion with the electronic receiver gain, which will be defined here as G_{rec}.

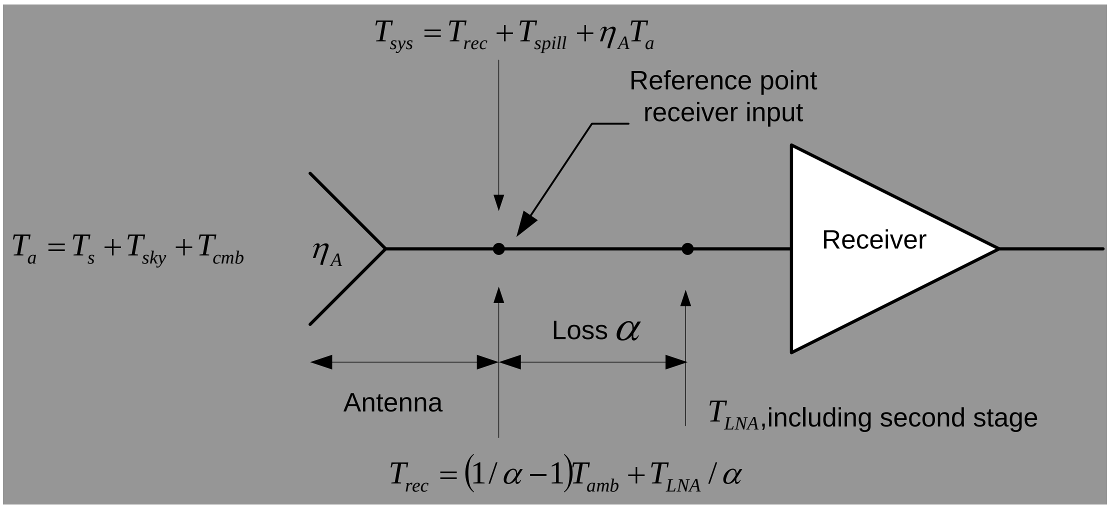


Figure 1. Definition of noise temperatures in a single-channel antenna/receiver system

3 Replacing an array system by an equivalent single-channel system

Fig. 2 shows a functional block diagram of a phased array receiving system, consisting of N elements of which the outputs are summed in a linear beam forming network. We will transform this system into an equivalent single-channel receiving system, following the steps outlined in [2], using the relevant formulas for various gain factors and noise temperatures, defined there. This section outlines the main steps in transforming an array system into an equivalent system with a single-output antenna and a two-port receiver; details may be found in [2].

3.1 Equivalent systems

The available signal power at the output of the beam forming network in Fig. 2 is described by:

$$S_o = P_o G_m \left| \sum_{n=1}^{N} \left(\sqrt{G_{en}}\right) a_n \exp(j\theta_n) \right|^2 ,$$

(1)

where P_o is the output power of a lossless isotropic antenna, G_{en} is the array antenna element gain, G_n is the available gain of a channel from the output of the n-th antenna element to the beam former output,

G_m is the maximum value of G_n, used for normalization and $a_n = \sqrt{\frac{G_n}{G_m}}$ is the effective amplitude taper of the n-th receiver channel transfer function. θ_n is the total phase shift of the *n*-th receiver channel with respect to that of the reference channel, accounting for beam steering and/or a phase taper.

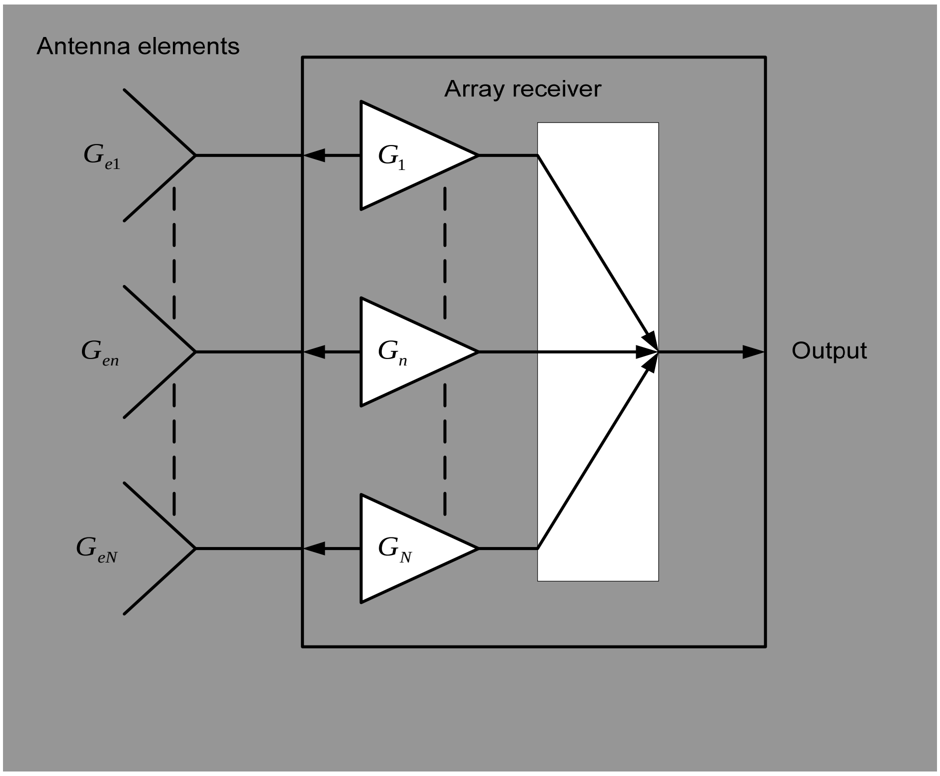


Figure 2. A general array receiving system

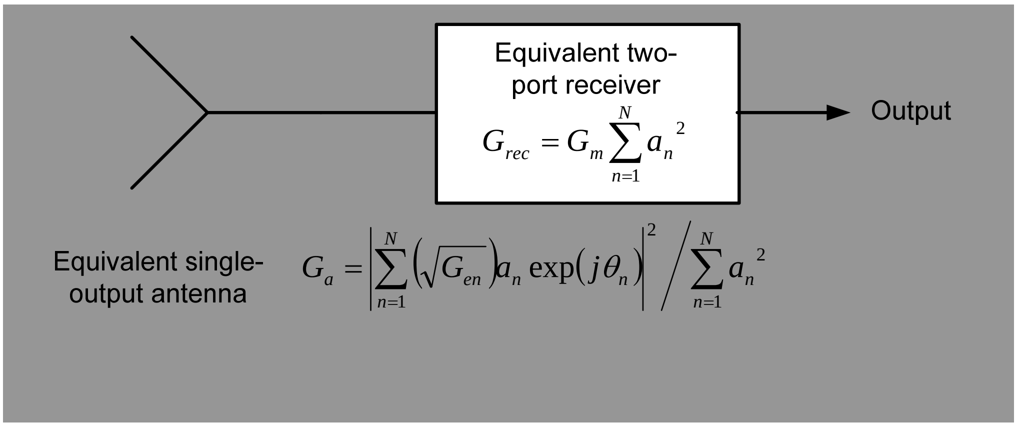


Figure 3. The equivalent receiving system

The power gain of an array antenna is defined by ([2]):

$$G_a = \left| \sum_{n=1}^{N} \left(\sqrt{G_{en}}\right) a_n \exp(j\theta_n) \right|^2 \Big/ \sum_{n=1}^{N} a_n^2 .$$

(2)

Substitution of (2) into (1) results in

$$S_o = P_o G_a G_m \sum_{n=1}^{N} a_n^2 ,$$

(3)

which shows that the array receiving system may be represented by an equivalent single antenna with output $P_o G_a$ and a two-port receiver with $G_{rec} = G_m \sum_{n=1}^{N} a_n^{\ 2} = \sum_{n=1}^{N} G_n$,

(4) as shown in Figure 3.

G_{rec} is the ratio of the receiver output power to the input power of a single channel when all array receiver inputs are excited by uncorrelated sources of equal power and is called the uncorrelated receiver gain, which will be used as the definition of receiver gain for a phased array. The antenna gain G_a accounts for the phasor summation in the array beam former of signals from the antenna elements.

Considering an ideal system, using a noiseless passive combiner as the array receiver, then $G_{rec} = 1$ and the array receiving system is equivalent to a conventional antenna with antenna gain G_a. The antenna gain of an array therefore includes the summation with a lossless combiner, including phase and amplitude weighting, fully defining the array beam properties and thus those of the equivalent single antenna. The receiver gain accounts for gains and losses in the receiver system, other than those for the antennas.

3.2 Definition of array noise temperature

The equivalent two-port receiver with gain G_{rec} may be used to define the effective input noise temperature of the array receiver, which is given by $T_{rec} = \dfrac{N_o}{kG_{rec}}$,

(5) where N_o is the excess output noise density of the array receiver, only due to receiver noise and k is Boltzmann's constant. N_o may be calculated from the summation of the product of the effective input noise temperature T_n and available gain G_n of individual receiver channels in the array environment, giving

$$T_{rec} = \frac{\sum_{n=1}^{N} G_n T_n}{G_{rec}} = \frac{\sum_{n=1}^{N} G_n T_n}{\sum_{n=1}^{N} G_n} .$$

(6)

It is emphasized that G_n and T_n are defined here as the effective available gain and noise temperature of individual receiver channels. In the case of arrays without coupling between antenna elements, noise emanating from individual receiver inputs will not couple to other receiver inputs and the equivalent gain and noise temperature are equal to the values for an isolated antenna channel. Formula (6) then gives a proper description of the array receiver noise temperature. However, in aperture and focal plane arrays, in general there may be considerable coupling between antenna elements. Noise emanating from individual receiver inputs will then couple into other receiver channels and add up coherently in the beam former. The resulting effective channel noise temperature and available gain may be described with the conventional two-port formulas for noise temperature and available gain, but with the source (antenna) reflection coefficient replaced by the active reflection coefficient or impedance for that channel, depending on a particular array excitation ([3]). By taking the (noise) coupling effect into account in this way, the array may be considered as an array without coupling between the antenna elements, but with modified available gain and noise temperature. Therefore, also in the case of noise coupling, formula (6) gives a proper description of the array receiver noise temperature.

Related to the array receiver, T_{rec} is the noise temperature of uncorrelated noise sources with identical power at the outputs of the antenna elements. To obtain the total output noise, the contribution from antenna related noise (T_{spill} and T_a, as described in section 2 for a conventional receiver) should be included by adding the effective noise temperature of the array to T_{rec}.
Antenna related noise finds its origin outside the array and is a function of the antenna pattern and power gain and may be expressed as: $T_x = \frac{1}{2k}\int G_a(\theta,\varphi)w(\theta,\varphi)d\Omega$, where $w(\theta,\varphi)$ is the spatial distribution of power density (W/Hz/steradian) and G_a is the power gain. The factor $\frac{1}{2}$ accounts for the fact that only one polarization is received by the antenna. With the definition of the power gain of the array antenna, T_x becomes the external noise temperature at the input to the equivalent receiver and includes all antenna related noise contributions as defined in section 2. Its contribution to the output noise density is thus kT_xG_{rec}. The total output noise density is then $k(T_x + T_{rec})G_{rec} = kT_{sys}G_{rec}$, with T_{sys} the system temperature of the array receiving system, defined at the input of the equivalent conventional receiver.
The above shows that the system noise temperature, and thus the output signal-to-noise ratio, of an array receiver may be computed in the same manner as for a conventional receiver, which is a direct consequence of the definition of T_{rec}.

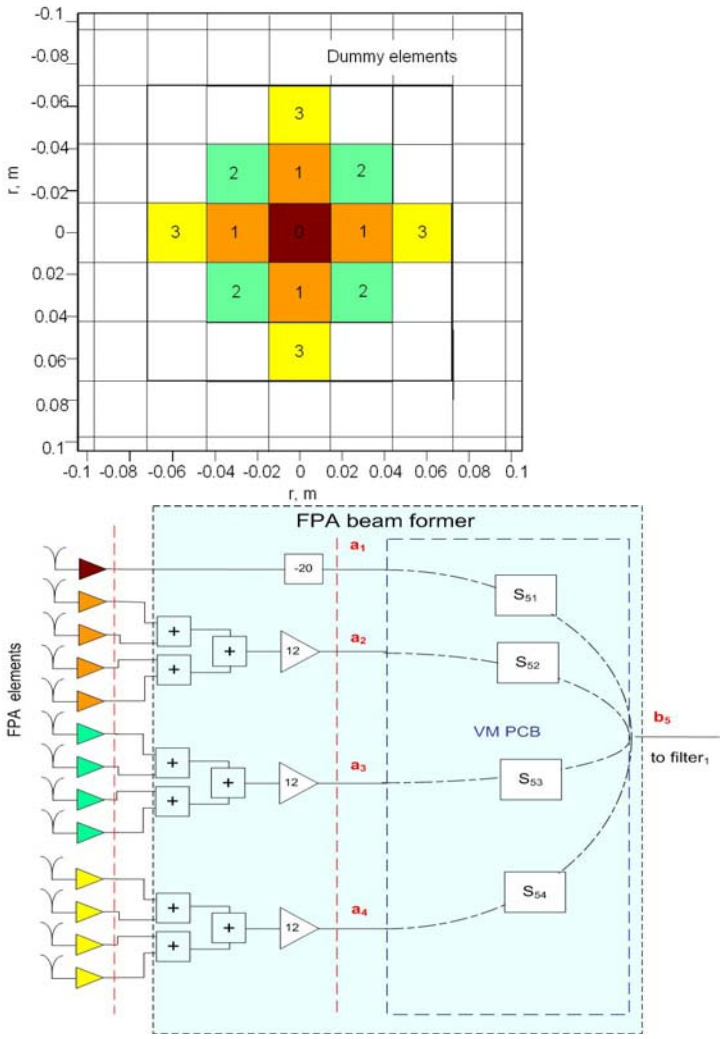
4 Calculation of equivalent noise and gain for a phased array

The noise temperature of an array receiver may be computed by determining the excess output noise and applying the definition (5) or using (6). Although, at first glance, this may look rather complicated for an array receiver with several sections and levels of combiners, the array receiver may be simplified to a set of identical two-ports with combiners, which may include an amplitude taper. The effect of coupling between array elements should be taken into account in the calculation of available gain and noise temperature of individual channels. Because the receiver gain and noise temperature are then defined using uncorrelated sources, a phase taper in the combiner can be ignored for the following discussion.
Let us first consider the simple N-channel array system of Figure 2, consisting of sets of identical two-ports and one combiner with amplitude taper. The cascade of two-ports in each channel is represented by one equivalent two-port, giving a set of N identical two-ports, having a gain G and noise temperature T. The gain of an equivalent two-port receiver is then given by (4): $G_{rec} = \sum_{n=1}^{N} G_n = G\sum_{n=1}^{N} G_{cn} = GG_c$, where G_{cn} is the gain of the n-th input port of the combiner to the output and G_c is the uncorrelated gain of the combiner.
Assuming that the noise of the two-ports and the combiner is uncorrelated, the excess output noise density is: $N_o = kTG_{rec} + kT_0(1 - G_c)$,
(7)
in which the first term is due to the channel two-ports and the second term is due to the combiner. Using (4), (5) and (7) leads to the noise temperature $T_{rec} = T + \frac{T_0}{G}(\frac{1}{G_c} - 1)$, which

represents the cascade noise formula for the equivalent two-port receiver, consisting of one channel two-port and a (second-stage) passive lossy two-port with gain G_c, representing the combiner.
In the analysis it was assumed that the amplitude taper is incorporated in the combiner. When equally coupled combiners and sets of two-ports are used, the taper may be obtained by a set of taper pads across the array at a point in front of the combiner input ports. The set of taper pads may then be represented by a single passive pad, with gain equal to the mean square voltage taper $\overline{a_n^2}$.
A more complicated array system with several sets and levels of combiners may be divided in simple array units as described above, which are then replaced by single two-port receiver sections. Using this procedure a complex array system may be simplified in repeated steps to obtain the analyzed basic N-channel array system. In conclusion an array system may be represented by an equivalent receiver consisting of a cascade of two-ports, for which the gain and noise temperature can be calculated with the conventional formula for a cascade of two-ports. A more detailed description is given in [2].

5 Example for the FARADAY array

Figure 4a shows a schematic top view of the Vivaldi FARADAY Focal Plane Array ([4]), with the 13 active elements arranged in three rings and a central element. In Figure 4b a block diagram of the amplifiers and beam former arrangement is shown, with vector modulators giving the relative weighting of the central element and the rings with respect to each other. The right hand figure comprises three levels of beam forming, which will be simplified using the procedure discussed in section 4, to calculate the equivalent two-port receiver gain and noise temperature. Although this procedure appears to be tedious, it shows in some detail the various steps that have to be taken to calculate array noise temperature and gain, according to the approach described in this document and underlines its usefulness. This calculation procedure has been implemented in an Excel spread sheet.
The array and beam former configuration of the FARADAY array is given here just as an example how to apply the procedure from section 4 to calculate the equivalent gain and noise temperature of a complicated FPA, when the properties of the individual array components are known. For that purpose gain and noise temperature of individual amplifiers are assumed, which are not necessarily equal to the values in the FARADAY system. Therefore the results do not exactly represent the present performance of the FARADAY array.



a) **b)**

Figure 4. Top view of the Vivaldi FARADAY FPA arrangement (a) and a schematic diagram of the 2005 beam former configuration (b)

5.1 Gain and noise temperature of the equivalent two-port for the rings

As shown in Figure 4, the first level of beam forming comprises three identical sets of amplifiers and beam formers for the three rings. Each ring consists of four low noise amplifiers of which the outputs are combined in a 4-way combiner. The gain and noise temperature of the individual components are given in the schematic diagram of one ring in Figure 5. According to section 4 the ring may be replaced by an equivalent two-port amplifier, followed by a passive lossy two-port with a gain that is equal to the uncorrelated gain of the combiner (-1 dB). This gain is calculated from the loss in the individual branches of the combiner (-7 dB for a 4-way combiner, including 1 dB insertion loss) and the combination of four equal input powers with equal weights, which increases the total output power with 6 dB with respect to a single input. In the calculation of the noise contribution of the rings, the noise contribution of the lossy two-port should be taken into account as the second stage contribution from an attenuator at ambient temperature.
The 12 dB amplifiers subsequent to the rings may be added to the equivalent two-ports directly, forming a chain of three two-ports, which may be further simplified by replacing it by one single two-port with identical gain and noise properties. The final result is one two-

port with a gain of 37 dB and a noise temperature of 132 K (in this case the second stage contribution is only 2 K), which forms the basic element for the next level of beam forming. In the calculation of the equivalent input noise temperature of the rings and the central element the LNA noise temperature under noise matched conditions has been taken (130 K) as a starting point. In the array configuration also the effect of noise coupling, described by the active reflection coefficient ([3]), has to be taken into account as a change in effective noise temperature with respect to the minimum noise temperature. On top of that the FARADAY array configuration, with a number of dummy and cross-polar elements loaded with 50 ohm loads at ambient temperature, leads to an extra receiver input noise contribution. The coupling between the antenna elements is estimated to give approximately 20 % efficiency loss of the FPA, introducing a noise contribution of 60 K at individual receiver inputs. In the calculation of the FPA receiver noise temperature the noise coupling contribution and the ambient coupled noise simply add to the amplifier noise. To keep the calculations simple and straightforward for this example, these contributions will be neglected in the following calculations. By taking them into account in the noise temperatures of the individual channels, their effect on the equivalent array

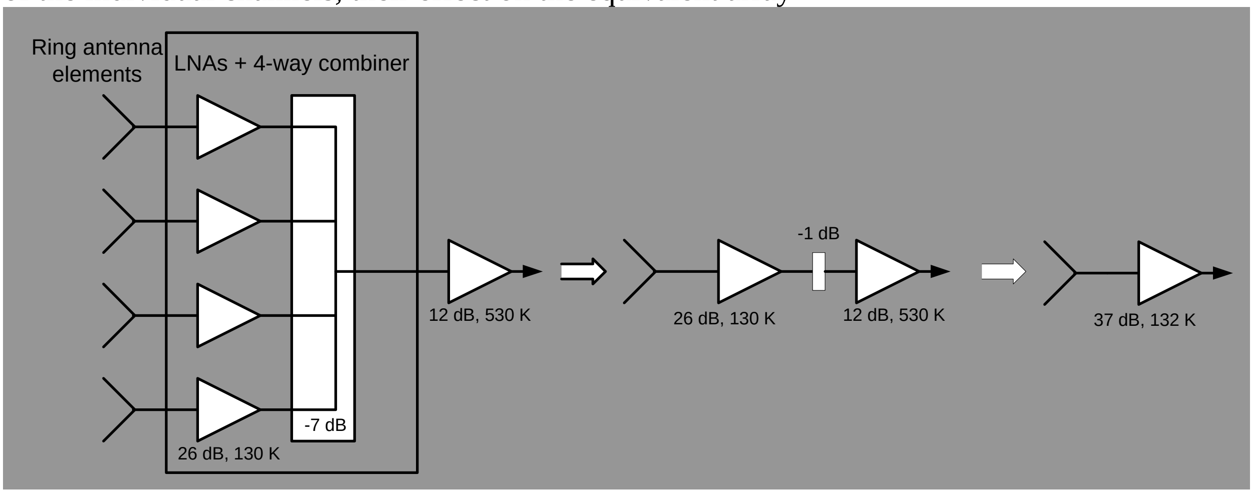


Figure 5. Transforming a ring system into an equivalent two-port receiver

noise temperature may be easily calculated at a later stage, e.g. using the spread sheet program mentioned in the introduction of this section.

5.2 Combining the rings

The next level of beam forming, shown in the schematic diagram of Figure 6, comprises a section with the equivalent two-ports defined in section 5.1, vector modulators giving the weighting factors and a 3-way combiner. In this case the power levels at the combiner inputs are not the same, but are lower by 4 dB and 7 dB with respect to the maximum level. This effect may be modeled by incorporating the relative weighting coefficients in the beam former and replacing the vector modulators by 20 dB attenuators at ambient temperature (It is assumed that the vector modulators do not show excess noise with respect to passive attenuators at ambient temperature). The equivalent two-ports with equal gain of 37 dB are now replaced by new identical two-ports with 17 dB of gain and a noise temperature of 138 K (including 6 K contribution from the 20 dB attenuator).

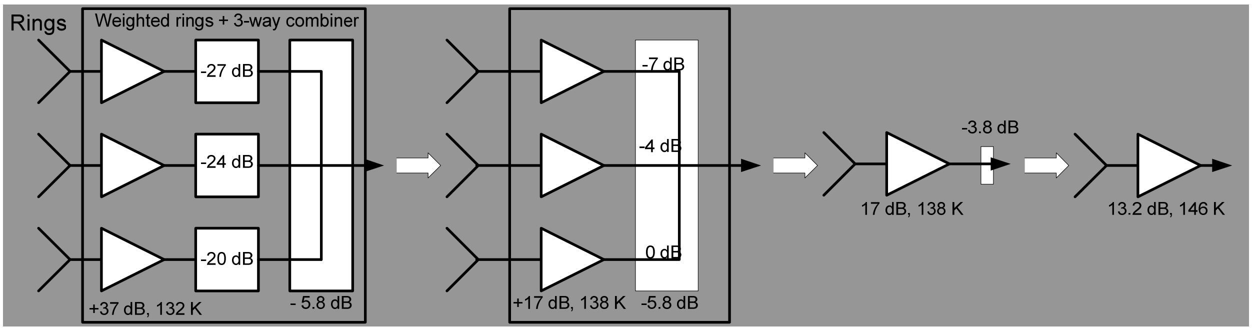


Figure 6. Combining the weighted rings

Similar to the procedure in sections 4 and 5.1 the rings are combined to a new equivalent two-port with the same properties, followed by a lossy two-port with the uncorrelated gain of the 3-way combiner including the weighting factors. The uncorrelated gain may be calculated as -3.8 dB by taking the loss of a single path in the combiner (-5.8 dB) and adding relative powers of the three inputs (+2 dB). As a last step, similar to that in section 5.1, an equivalent two-port may be defined as an element for further calculation. This two-port has 13.2 dB gain and a noise temperature of 146 K, including the contribution from the combiner.

5.3 Adding the central element

In the final combiner the output of the equivalent two-port defined in section 5.2 will be added to the output of the central element. The branch with the central element, shown in Figure 7, consists of a first stage LNA with the same properties as the LNAs in the rings (gain 26 dB, noise temperature 130 K), followed by a 20 dB attenuator and a directional coupler with 1 dB insertion loss, giving a net gain of 5 dB. This cascade of two-ports may be replaced by an equivalent two-port with a gain equal to that of the two-port of the combined rings (13.2 dB), followed by an attenuator of 8.2 dB representing the weighting of the central element with respect to the combined rings. The equivalent noise temperature of the central element two-port is calculated as 143 K, taking into account the LNA noise and the second stage contribution of a 12.8 dB attenuator.

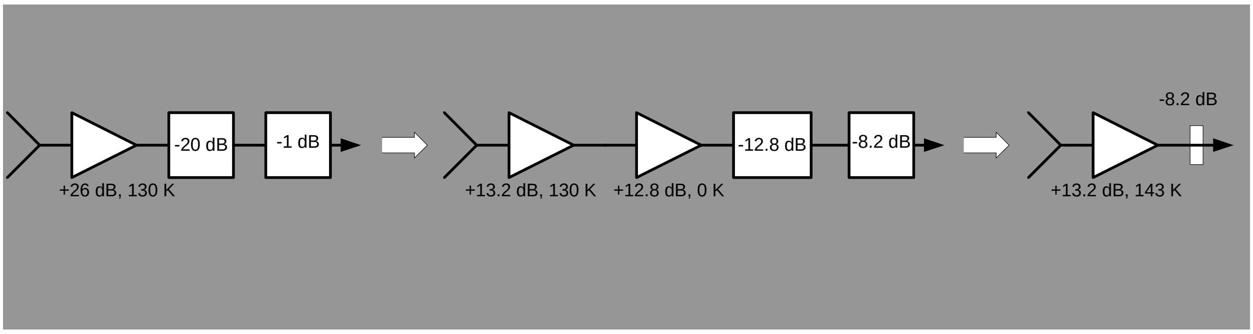


Figure 7. The equivalent central element

The final step is to add the central element to the combined rings, as shown in Figure 8. The gain of the overall FPA system is determined in a similar way as in the previous sections, by calculating the uncorrelated gain of the combiner (-3.4 dB), giving a total gain of 9.8 dB. The noise temperature of the equivalent amplifier is calculated as the sum of the noise temperatures of the combined amplifiers, weighted with their gain and results in 146 K. To calculate the overall FPA noise temperature, a 16 K contribution of the 3.4 dB attenuator should be added, resulting in an effective FPA input noise temperature of 162 K.

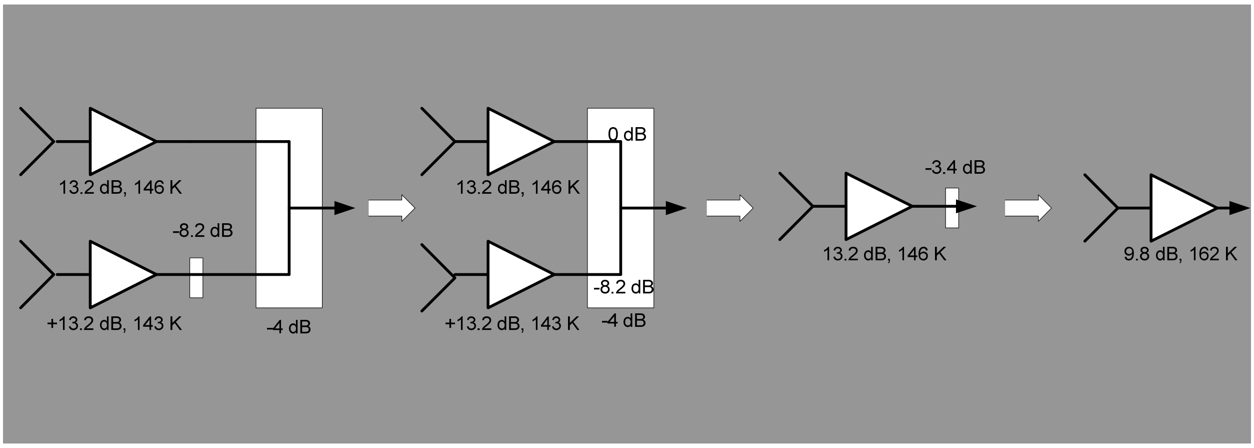


Figure 8. Adding the central element to give the equivalent FPA receiver

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