Memo 78

The SKA as a Survey Instrument-Implications for the Antenna Size

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The SKA as a Survey Instrument - Implications for the Antenna Size

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1 Introduction

The SKA will be used mainly as a survey telescope at all of its wavelengths, much like the Sloan Telescope has been used. This is clear from the key science cases as well as the more detailed discussions in the Science Document. It is also clear from the specification calling for large fields of view at all wavelengths. A field of the order of one degree can be obtained with a single dish near 20 - 30 cm, but that diminishes at the shorter wavelengths. Yet surveys will need to be made at the shorter wavelengths, whether it be for transients at short cm wavelengths, free-free emission from OB clusters in the Milky and other galaxies, Zeeman measurements in dense gas using transitions of CCS, or the search for red shifted CO in the EOR at z $\geq$ 6. This means that the large fields will be obtained from multiple pointings of the array antennas, and it makes sense to consider this circumstance in the optimization of the array. If we look back at the uses of telescopes, we often find that the most interesting results come from surveys. This is clear from the citations, if nothing else. Our discussion somewhat follows Sandy Weinreb's memo of May 14 (2006), but we draw a different conclusion.

For multiple pointing observations with an array of n antennas of diameter D, the point source sensitivity of the multiple pointing survey is proportional to nD, not nD^2 as it is for a single pointing of the array. We consider this for the optimization here. We do not include focal plane arrays because they provide poor bandwidth or poor receiver noise temperatures or both. Including them for a small part of the overall range does not help very much in any case, since the science case demands survey capability at all wavelengths. These comments are particularly relevant for the upper part of the band which extends from 1 GHz to 25 GHz. The cost of optical fiber over large distances will eventually be an issue in the optimization and we will come back to it later.

2 Survey Sensitivity

An array of n antennas each of diameter D and with single on-axis feeds has a point source sensitivity proportional to nD^2 for a single pointing of the array. It is sometimes overlooked that if multiple pointings are used, as in the case of a survey, the resulting sensitivity for any point source that is detected is proportional to nD. It is due to the fact that with larger diameter antennas the antenna solid angle decreases with an increase in antenna area and a smaller fraction of a given time can be devoted to the observation of any one pointing (cf,Sargent and Welch, 1993; Thompson et al, 2000; Bregman, 2004).

The argument follows from the basic sensitivity equation for an array (Thompson, et al, 2000). The RMS flux noise for an observation of duration t, with n antennas of diameter D and aperture efficiency η_a, and with receiver bandwidth $\delta\nu$ and system temperature T_s is

$$\sigma_n = \frac{2kT_s}{n(\pi/4)D^2\eta_a\sqrt{\delta\nu t}} \propto 1/nD^2 \tag{1}$$

If the region of study, Ω_r is larger than the primary beam, Ω_b, multiple pointings must be made to cover it; in the same total observing time t, less time is spent on each pointing. $4\Omega_r/\Omega_b$ pointings at half beam spacings are needed. This number is the reciprocal fraction of the total time spent on each pointing. The primary beam size, Ω_b, is $\pi\lambda^2/4D^2$. Substituting, we find that the noise for any point source detected in a survey of the region Ω_r is

$$\sigma_n = \frac{32\sqrt{\Omega_r}kT_s}{\pi^{1.5}nD\lambda\eta_a\sqrt{\delta\nu t}} \propto 1/nD \tag{2}$$

3 Cost Optimum Choice for Antenna Diameter

The principle cost items are the antennas and the antenna electronics. Because there are $n^2/2$ correlations for n antennas, the correlator cost is also a possible issue. However, historically the correlator cost usually turns out to be only 5 - 10% of the total array cost because of advances in digital electronics during the development phase, and we ignore it here. Empirically it has been found that the antenna cost can be represented as a power law of the diameter over a fairly large range of diameter. With A as the unit cost for the electronics and n the number of antennas, we can write the overall cost as

$$Cost = nA[1 + (D/D_o)^\alpha] \tag{3}$$

nA is the cost of the electronics, and D_o is the antenna diameter for which the antenna cost is the same as that of the electronics. We examine two choices for the cost optimization. The first is the single pointing sensitivity, where S = cnD^2. c is a constant. Substituting for n above, we find

$$Cost = \frac{SA}{cD^2}[1 + (D/D_o)^\alpha] \tag{4}$$

To find the minimum cost for a given sensitivity, we differentiate with respect to D, and set the result equal to zero. We find

$$(D/D_o)^\alpha = \frac{2}{\alpha - 2}, or (D/D_o) = (\frac{2}{\alpha - 2})^{1/\alpha} \tag{5}$$

The second choice is the survey case, where S = c'nD. Substituting this into equation (3) above gives the following cost equation.

$$Cost = \frac{SA}{c'D}[1 + (D/D_o)^\alpha] \tag{6}$$

Differentiating this expression with respect to D and setting it to zero gives us

$$(D/D_O)^\alpha = \frac{1}{\alpha - 1}, or (D/D_o) = (\frac{1}{\alpha - 1})^{1/\alpha} \tag{7}$$

The table shows values for these quantities for three typical values of the parameter α that are often used.

α	2/(α-2)	1/(α-1)	(D/D_o) (sng)	(D/D_o) (svy)
2.5	4	0.7	1.7	.87
2.7	2.9	0.6	1.48	.83
3.0	2.0	0.5	1.26	.79

The first column is the exponent α. The second is the ratio of the antenna cost to electronics cost for the single pointing case. The third column is the same ratio for the survey case. The forth column is the single pointing ratio (D/D_o). The last column gives that ratio for the survey case.

In his memo, Sandy adopts $\alpha = 2.7$ and 12m as being optimal for a single pointing application such as for the DSN which will generally be following one space craft or several subcrafts in the neighborhood of one planet in a field smaller than the 12m beam. For doing multiple missions which are in very different directions, probably subarrays will be the choice for the DSN.

For the SKA whose science is heavily weighted toward surveys the second optimization is the relevant case. That is, it is not nD^2/T that should give the optimum, but rather nD/T. For the same estimates of antenna costs used to obtain 12m as the optimum in the first case, we get a diameter closer to 6 or 7m for the survey case.

4 Discussion

If we insist on having a baseline antenna diameter at this stage in the planning, there is a better case for a 6 - 7m antenna than for the 12m. It is enlightening to consider some round numerical values for a comparison. Compare the 12m with a 6m for both the single pointing and a survey. Following Sandy we take the receiver cost to be 30k\$, the cost of a 12m antenna to be 169k\$, and the cost of a 6m antenna to 30k\$. For the single pointing case we suppose 5000 12m antennas, and it would take 20,000 6m antennas to provide the same area. The price for the 12m array is 995K\$, and for the 6m array it is 1,200K%, 20% more expensive. Now for the survey case, we make nD the same in both cases, keeping the number of 12m antennas the same. This time only 10,000 6m dishes are needed. Then the 6m cost is 600K\$, and the survey sensitivity is the same. This 6m array will have the same sensitivity as the 12m array for the bulk of the science and will cost only 60% as much.

For single pointing observations, including some of the follow up studies, which will occupy a small fraction of the observing time, more integration time than is devoted to each survey pointing may be used. If this integration time were greater by a factor of four, the original single pointing sensitivity would result in the example discussed above. More importantly, the survey sensitivity will be optimum and at a much lower price.

Another way of minimizing cost is to use a single wide-band feed on the optical axis for as much of the whole frequency range as possible from, say, 0.5 to 25 GHz.

The fiber costs will probably be the biggest problem for the distant stations. If we kept the same number of these stations for the smaller antennas, the single fiber runs would be the same. Having more smaller antennas at the station would produce a better station beam which would be an advantage. At the same time, the best way to get the largest beam for the station would be to use a single antenna there. This is probably the best way for the station, and the smaller antenna will give a larger beam.

It is time to rethink the SKA antenna specifications from memo 45 which have been set in terms of A/T $\sim nD^2$/T. The more reasonable specifications should be set in terms of nD/T.

5 references

- Thompson, A. R., Moran, J. M., Swenson, G. W., 2000, *Interferometry and Synthesis in Radio Astronomy*, New York, Wiley.

- Sargent, A. I., Welch, W. J., 1993, *Millimeter and Submillimeter Interferometry of Astronomical Sources*, Ann. Rev. Astron. Astrophys, 31. 297.

- Bregman, J. *System Optimization of Multibeam Aperture Synthesis Arrays for Survey Performance*, 2004, Experimental Astronomy, 17, 365.

- Weinreg, S. 2006. USSKA Memo 41.