SKA Correlator Input Data Rate

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Introduction

One of the major costs in the SKA is data transport from the antenna to the central beamformer or correlator. In this memo formulae are derived that allow all the SKA concepts to be compared. It is found that data rate per steradian does not depend on the size of the antenna. For a given performance factors that do affect the data rate are T_{sys}, wavelength, sensitivity, the use of mosaicing and beamforming with arrays of antennas.

Filled Aperture Antenna

The simplest case to consider is a full sensitivity SKA composed of a number of filled-aperture antennas, where the signal from each antenna is transported to the correlator. For the moment assume the antenna is circular with effective area A_{eff} m^2. The effective diameter D_{eff} of the antenna, in metres, is $\sqrt{(4A_{eff}/\pi)}$ and the beam width is approximately $1.09\lambda/D_{eff}$ radians [1 Appendix C]. The field of view FOV can now be calculated

$$\begin{aligned} FOV &= \frac{\pi}{4}\frac{1.09^2\lambda^2}{D_{eff}^2} \\ &= \frac{\lambda^2}{A_{eff}}\left(\frac{1.09\pi}{4}\right)^2 \\ &= 0.73\frac{\lambda^2}{A_{eff}} \text{ steradians} \\ &= 2{,}400\frac{\lambda^2}{A_{eff}} \text{ square degrees} \end{aligned} \qquad (1)$$

The final form of this relationship holds even for non circular apertures such as an ellipsis for LAR [2] and rectangles for cylindrical reflectors [1]. The number N of these antennas needed to satisfy a sensitivity requirement of S m^2/K is

$$N=S.\ T_{sys}/A_{eff} \qquad (2)$$

If the data rate per beam per antenna is B bits/s then the total data rate into the correlator is simply the data rate per antenna multiplied by the number of antennas. The data rate per steradian R is then found by dividing by the field-of-view.

$$
\begin{aligned}
R &= \frac{B.N}{FOV} \\
&= \frac{S.T_{sys}B}{A_{eff}}.\frac{A_{eff}}{0.73.\lambda^2} \\
&= \frac{1.37S.B.T_{sys}}{\lambda^2} \text{ bits/s.steradians} \\
&= \frac{1.37A_{te}.B}{\lambda^2} \text{ bits/s.steradians}
\end{aligned} \qquad (3)
$$

where B is the data rate per beam per antenna in bits/s, λ is the observing wavelength in metres, and $A_{te} = N.A_{eff}$ is the total effective area in m^2.

It is seen that there are two equivalent equations for the data rate per steradian and neither depend on the effective area of the individual antennas. Of the two the second is the simpler understand. If there was a single antenna then the factor λ^2/A_{te} is approximately the FOV and B is the bit rate and it seen that equation 3 holds. If the area was divide between N antennas then the total bit rate increases by N. This is compensated by an increase in the FOV of the antenna by the same factor. Thus the bit rate per steradian stays constant. In the alternative equation the total effective area A_{te} is be replaced by $S.T_{sys}$

The above assumes full sensitivity and only a single beam from each filled aperture antenna, but the result still holds for multiple independent beams. In practice the SKA sensitivity can vary and

- Multiple beams may be used to form a mosaic or
- The beam may be formed by summing the signals from an array of smaller antennas

Sensitivity

In equation 3 it is seen that the data rate is directly proportional to sensitivity. An instrument with half the sensitivity has half the data rate but the instrument has degraded performance. Thus the data rate given in equation 3 is not a fair comparison of SKA concepts with different sensitivities. A good example of what happens when sensitivity changes is the Luneburg lens concept designs. In the original white paper [5] a full sensitivity instrument with one beam at any one frequency was proposed. In the update [6] the design was changed to a four beam instrument with half the collecting area and half the sensitivity. With half the area the integration times for a given sensitivity had to increase by a factor of four. By having four independent beams the total available integration time is increased by four. Thus over a large number of observations the updated instrument can achieve the same sensitivity in the same time as the original instrument. In the updated instrument there are four beams each with half the total data rate of the original design making the total data rate double that of the original design.

In general if the sensitivity decreases by a factor k then the integration time must increase by k^2 to compensate. Thus the area of sky that can be observed per day to a given sensitivity is decreased by $1/k^2$. In this sense the decrease in sensitivity has the same effect as a decrease in the effective FOV. Alternatively the number of beams or total FOV could be increased by k^2 to compensate. This increase causes the data rate to increase by k^2 but leaves the total area of sky observed per day unchanged. If S_{ska} is the full SKA sensitivity, the increase in the number of beams or data rate needed to compensate is $(S_{ska}/S)^2$. Including this term in equation 3 gives a sensitivity normalised data rate C

$$C = \frac{1.37.S_{ska}{}^2.B.T_{sys}}{S.\lambda^2} \text{ bits/s.steradians} \qquad (4)$$

This sensitivity normalised data rate can be used to calculate an effective data rate for concepts whose sensitivity varies with elevation angle such as LAR [2] and the aperture array [3]. In these concepts the sensitivity varies as cos(zenith angle). The average data rate can be found by averaging the reciprocal of this zenith angle sensitivity dependence over the available sky. For LAR where the antenna can scan down to a zenith angle of 60 degrees the sensitivity normalised data rate over the full sky increases by 1.4 compared to the data rate at zenith. For the aperture array where the scan angle may be limited to 45 degrees the rate is increase by a factor of 1.19.

For the cylinder [1] there is a foreshortening loss in only one dimension and the data rate increases by 1.25 if observations are made down to a meridian distance of 60 degrees. However, as all declinations are available at transit without any loss of sensitivity it is expected that observing will preferentially be done near transit. If the majority of observations occur within 30 degrees of transit then the data rate increase is limited to 1.05.

Mosaic

When a single beam from an antenna is processed, only the data within the half power beam is normally used. In contrast when a mosaic is made, all the data within the primary beam becomes useable. To estimate the effect of this on data rate, consider the simple case of a one dimensional mosaic as might occur with data from a cylindrical reflector where the beams overlap at the half power points. If the noise in the beams is uncorrelated then the sensitivity relative to the peak is 0.7 at the overlap point. The average sensitivity is 0.861 of the peak value, for a Gaussian beam. The variation in sensitivity can be reduced further by observing on a finer grid of points. In the limit, sensitivity approaches 0.868 of the peak value if the total observing time is left unchanged. By comparison, the average sensitivity in a one dimensional cut across a single Gaussian beam is 0.81 of the peak sensitivity. Thus mosaicing along one dimension improves the average sensitivity by 7%.

A set of one-dimensional mosaics can then be mosiaced in the orthogonal directions. This is equivalent to two dimensional mosaicing on a rectangular grid. After doing this the average sensitivity is $0.868^2 = 0.753$ of the peak sensitivity of an equivalent single beam observation. Here, the equivalent single beam observation is one of a set of observations over the same area done on a rectangular grid with a spacing equal to the half power beam width. In the mosaic observation a much finer grid covering the same area is used with the total observing time kept the same. The sensitivity of an equivalent single Gaussian beam observation average over the beam down to the half power point is 0.721. Thus the mosaic has a 1.044 sensitivity advantage. This sensitivity advantage reduces survey time for a given sensitivity by a factor of 1.09.

But, in isolated single beam observations the area imaged is circular and for a mosaic the equivalent area is square. This gives the mosaic a 4/π= 1.273 area advantage. Thus a mosaic reduces the sensitivity normalised data rate by a factor of 1.39 relative to observing with isolated beams.

The above argument assumes that noise in the overlapping beams is uncorrelated. This may not be true if the beams are formed at the same time from a focal plane array or aperture array. In this case the noise in the beams is correlated and there may not be any improvement in the sensitivity when data from adjacent beams are added. However, if the beams do not overlap by more than the half power points the results will be no worse than an accumulation of single beam images. In this case the sensitivity normalised data rate is the same as that of a single beam. In some cases it might be desirable to image instantaneously with beams spaced at less than a beam width. This allows low frequency spatial data to be recovered and increases average sensitivity at the expense of increased data rate and correlator capacity.

For example in the LAR concept [2] it is proposed that the image be sampled with beams spaced λ/2D apart. This increases the data rate by a factor of four and increases the sensitivity for a Gaussian beam to 0.893 when averaged over a square area of width equal to half a beam width. Compared to averaging over all data within the half power points of the beam this is a 24% increase in average sensitivity. The increase in sensitivity is achieved without changing the size of the area imaged so the sensitivity normalised data rate is reduced by the square of the sensitivity change. But to achieve this, the data rate was increased by a factor of four. Thus the sensitivity normalised data rate increases by a factor of $4/1.24^2 = 2.6$ compared to single beam observations.

Antenna Arrays

Some concepts group a number of small area antennas into arrays [4, 5]. This allows point source sensitivity to be increased without increasing the data rate, but decreases the FOV when compared to a filled aperture antenna with the same effective area. This will result in an increase in the data rate per steradian into the correlator. To calculate the decrease in beam size consider an array of small antennas with a total effective area $A_{eff.}$ As a single filled aperture antenna the actual diameter D would be $\sqrt{(4A_{eff}/\pi\eta)}$ where η is the aperture efficiency. If the array is formed on a hexagonal grid and there is no blockage down to elevation angle of θ then the effective diameter D_a of the array [1 Appendix C] is increased by a factor of $\sqrt{1.1}/\sin(\theta)$:

$$D_a = \sqrt{\frac{4.4A_{eff}}{\pi\eta}}\frac{1}{\sin(\theta)}$$

With a uniform grading across the array the beam width at zenith φ_z is

$$\begin{aligned}\varphi_z &= 1.02\lambda/D_a \\ &= \frac{1.02\lambda\sqrt{\pi\eta}.\sin(\theta)}{\sqrt{4.4A_{eff}}}\end{aligned}$$

and the field of view FOV_a is

$$FOV_a = \pi\varphi_z^2/4$$
$$= \frac{\pi.1.02^2\lambda^2\pi\eta.\sin(\theta)^2}{4.(4.4A_{eff})}$$
$$= \frac{0.58.\lambda^2\eta.\sin(\theta)^2}{A_{eff}} \text{ steradians}$$
$$= \frac{1{,}900.\lambda^2\eta.\sin(\theta)^2}{A_{eff}} \text{ square degrees}$$

this differs from the result in (1) by the factor $0.79\eta.\sin(\theta)^2$ and the sensitivity normalised data rate for an array C_{array} at zenith is

$$C_{array} = \frac{1.72.S_{ska}{}^2.B.T_{sys}}{S.\lambda^2.\eta.\sin(\theta)^2} \text{ bits/s.steradians}$$

As the array scans away from zenith the array foreshortens and the field of view increases by 1/cos(z), where z is the zenith angle. If there is no shadowing there is no loss of sensitivity and the sensitivity normalised data rate becomes:

$$C_{array} = \frac{1.72.S_{ska}{}^2B.T_{sys}}{S.\lambda^2}\frac{\cos(z)}{\eta\sin(\theta)^2} \text{ bits/s.steradians}$$

Taking a representative zenith angle of 45 degrees, a minimum elevation of 15 degrees and an aperture efficiency of 0.65 then C_{array} is twenty times greater than the sensitivity normalised data rate for a concept using a filled aperture antenna. Increasing the minimum elevation before shadowing to 30 degrees reduces the data rate to 5.5 times that of a single filled aperture antenna.

Summary

The sensitivity normalised data rate C is given by

$$C = k \frac{1.37 S_{ska}{}^2 B.T_{sys}}{S.\lambda^2} \text{ bits/s.steradians}$$

where the value of k depends on a number of factors. These are listed in the table below for a number of different possible conditions.

Table 1 Relative change in correlator data rates as a function of different antenna types and operating modes.

Antenna type	Factor affecting data rate	k
Filled aperture antennas	Independent beams	1.00
	Cos(zenith angle) gain to 60°	1.44
	Cos(zenith angle) gain to 45°	1.19
	Cos(meridian distance) gain to 60°	1.25
	Cos(meridian distance) gain to 30°	1.07
	Mosaic uncorrelated noise per beam	0.72
	Mosaic beam spacing λ/2D noise per beam correlated 100%,	2.60
Antenna arrays	Independent beams. Zenith angle z, Min elevation θ and aperture efficiency η	$\frac{1.25 \cos(z)}{\eta \sin(\theta)^2}$

Conclusion

For filled aperture antennas the sensitivity normalised data rate per steradian into the correlator is proportional to $T_{sys}/(S\lambda^2)$ where λ is the wavelength in metres and S is the sensitivity. The sensitivity dependence can be used to derive the data rate for antennas whose gain varies with antenna pointing. Data rate is also affected by mosaicing as it can use all the data available per beam giving a small increase in overall sensitivity. The final factor that affects data rate per steradian is the arraying of antennas. This can have a major effect on the data rate as the beam size for an array is very much smaller than that of an equivalent filled aperture antenna.

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