Data Transmission Requirements for the SKA: Small-N vs. Large-N

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November 26, 2003

1 Introduction

There are many proposed concepts for the construction of the Square-Kilometre Array (SKA). These engineering solutions can be divided into two classes: those that have hundreds or thousands of stations ("Large-N") and those that have a smaller number, in the range of 30 stations ("Small-N"). Large-N arrays would be composed of a large number of small-diameter elements such as reflector antennas or Luneburg lenses. Small-N arrays could be constructed out of either large-diameter elements, such as the Large Adaptive Reflector (LAR), or out of clusters of small-diameter elements, or even phased arrays. This report will consider the data transmission bandwidths for both classes to see if one configuration has an advantage over the other.

2 Assumptions

A number of assumptions have been made to simplify this analysis.

1. The effective area, receiver temperature, spillover, etc. are the same for both cases.
2. A zenith angle range of 0–60° will be observed.
3. The RF signal bandwidth is the same in both cases.
4. "Small-N" stations could be realized with either a single very large reflector antenna fed by a focal-plane array or a large cluster of small elements. In either case, beamforming networks are used to produce station beams of sufficient number to completely map the desired field-of-view.
5. "Large-N" stations are composed of clusters of M-elements and that the outputs of these elements are combined in beamforming networks. The beamforming networks produce station beams of sufficient number to completely map the desired field-of-view. (In Section 4.4 the case without beamforming is considered.)
6. $M = 13$ as given in the US SKA Consortium White Paper, 2002.

7. The desired field-of-view (1 square-degree at 1.4 GHz) is equal to the size of the primary beam of a single element in a Large-N array.

8. An increase in observing time is equivalent to an increase in data rate, since more telescope could be constructed so that the observing is all done concurrently.

3 Variables

The symbols and variables along with commonly-assumed values are listed in Table 1.

The number of beams needed by a Small-N element to map one-square-degree (F_{SN}) is simply the ratio of the aperture area divided by the area of an aperture with a one-square-degree beam, namely a single Large-N element.

The number of beams for a Large-N station to map one square-degree is F_{LN}. To derive F_{LN}, we first consider the case of a *filled* aperture of the same area as the station-array of M elements. Such an array will require $\sim M$ beams to map the field-of-view of a single element in that array. The actual station is composed of M elements spaced apart to prevent one dish from shadowing another when pointed to large zenith angles. This results in a dilute aperture which is distributed over a larger area, and thus has a narrower beam than an equivalent filled aperture. A shadowing correction factor, k_{shad}, is multiplied with the number of beams for a filled aperture (M) to obtain the number of beams for a station composed of a distributed array of small apertures. For zenith angles to 60°, $k_{shad} \geq 4$ because the elements have to be at least two element-diameter apart to avoid shadowing.

The sampling factor (k_{samp}) is included in the table to account for the focal-spot spacing needed to extract short-spacing information from interferometer measurements [1]. If a single beam samples the desired imaging field, then the sampling factor is 1. For cases where a field-of-view is synthesized from a number of focal spots on the focal-plane array, then the focal spots (or equivalently the beams on the sky) must be spaced so that Nyquist sampling occurs. With a square sampling pattern the spacing is $\lambda/2D$, resulting in a factor of 4 increase in the number of beams assuming that the half-power beamwidth of a beam is λ/D.

The foreshortening factor (k_{fs}) is applied to flat apertures, such as the LAR or phased arrays, that suffer a geometric reduction in effective aperture due to foreshortening as the zenith angle is increased. In the worst case, for a maximum zenith angle of 60°, the effective area is reduced to half the value when pointing at the zenith. This is compensated for by building additional stations, this will increase the data rate into the correlator by a factor of two. (This is a "soft" specification since it could be argued that a sky-weighted average instead of a worst-case value be used instead.)

The final factor in the table is k_{uni}, the factor by which the data rate must be increased so that the sensitivity across a single-field map is uniform. Consider an array of small-diameter collectors, from which the outputs are fed directly into a correlator without any beamforming. This produces a map of the sky with the field-of-view of a single element. That field is tapered by the single-element beam pattern, reducing the sensitivity away from the field centre. Now contrast this with a mosaic map produced by an array of large elements, such as LARs: each beam that goes into the mosaic has *full sensitivity*, so the mosaic has nearly uniform sensitivity to the edge of the field-of-view. Therefore, an SKA composed of small collectors will require additional observations

Table 1: Symbols and Variables

Symbol	Value	Description
A_{cor}	10^6m^2	Total area of SKA
D_{SN}	200m	diameter of Small-N station
D_{LN}	12m	diameter of Large-N element
A_{SN}	$\pi/4 \times D_{SN}^2 = 31 \times 10^3\text{m}^2$	area of Small-N station
A_{LN}	$\pi/4 \times D_{LN}^2 = 113\text{m}^2$	area of Large-N element
N_{SN}	$A_{cor}/A_{SN} = 32$	number Small-N stations/SKA
N_{LN}	$A_{cor}/A_{LN} = 8900$	number Large-N elements/SKA
M	13	number Large-N elements/station
k_{samp}	3 or 4	sampling factor
k_{fs}	1–2	foreshortening factor
k_{shad}	4	shadowing factor
k_{uni}	≤ 4	factor for uniform sensitivity
F_{SN}	$A_{SN}/A_{LN} = 280$	number of Small-N beams per sq. degree
F_{LN}	$k_{shad} \times M = 52$	number of Large-N beams per sq. degree
B_{ele}		element output data rate
B_{stn}		station output data rate
B_{cor}		correlator input date rate

in order to produce maps with uniform sensitivity, and this is equivalent to an increase in data rate. The Large-N array observation could be considered as a "one-field mosaic" since additional pointings surrounding the field-of-view will also extract low-order spacing information. An estimate for k_{uni} can be made by considering an observation consisting of the central field surrounded by three additional fields, for a total of four observations, yielding $k_{uni} \leq 4$.

4 Data-Rate Calculations

These calculations look at the data rate at two places: at the output of the station (B_{stn}) and at the input of the correlator (B_{cor}).

4.1 One station-beam transmitted to correlator

The calculation in Table 2 is for the data rate to send back only one beam per station. The Small-N station beam is narrower than the Large-N beam. If the elements in the Small-N array are flat (such as LAR elements) then the data rate into the correlator (B_{cor}) must be increased by a factor k_{fs} because more elements have to be constructed to compensate for foreshortening.

Table 2: Data Rate for One Station-Beam

Small-N			Large-N		
B_{stn}	$=$	B_{ele}	B_{stn}	$=$	B_{ele}
B_{cor}	$=$	$N_{SN} \times k_{fs} \times B_{stn}$	B_{cor}	$=$	$(N_{LN}/M) \times B_{stn}$
	$=$	$N_{SN} \times k_{fs} \times B_{ele}$		$=$	$(N_{LN}/M) \times B_{ele}$
	$=$	$64\,B_{ele}$		$=$	$685\,B_{ele}$

Table 3: Data Rate for the Field-of-View of a Large-N Station Beam

Small-N			Large-N		
B_{stn}	$=$	$(F_{SN}/F_{LN}) \times k_{samp} \times B_{ele}$	B_{stn}	$=$	$k_{uni} \times B_{ele}$
	$=$	$280/(M \times k_{shad}) \times k_{samp} \times B_{ele}$			
	$=$	$21 \times k_{samp} \times B_{ele}/k_{shad}$			
B_{cor}	$=$	$N_{SN} \times k_{fs} \times B_{stn}$	B_{cor}	$=$	$(N_{LN}/M) \times B_{stn}$
	$=$	$32 \times 21 \times k_{samp} \times k_{fs} \times B_{ele}/k_{shad}$		$=$	$(8900/13) \times k_{uni} \times B_{ele}$
	$=$	$670 \times k_{samp} \times k_{fs} \times B_{ele}/k_{shad}$		$=$	$685 \times k_{uni} \times B_{ele}$

For this scenario it does not matter how the stations are configured: each station sends back the same amount of data. However, since there are fewer stations in a Small-N array, the amount of data delivered to the correlator is $\sim 1/11$ times that delivered to the Large-N correlator.

4.2 Field-of-View of one Large-N station beam

In this comparison the field-of-view is the same for both configurations: it is the field within one station-beam of a Large-N station. This beam is produced by making a phased array from the M-elements in the station cluster and sending a single beam back to the correlator. Since Small-N stations have narrower beams (because the station diameter is greater), multiple overlapping beams need to be sent back to the correlator, and the sampling factor has to be applied. The calculations are summarized in Table 3. Note that the shadowing factor (k_{shad}) appears in the left column (Small-N) because the Large-N station beam size has been reduced, so fewer Small-N beams are required to map the field-of-view.

The data rate from a Small-N station is $21 \times k_{samp}/(k_{shad} \times k_{uni}) \sim 5$ times that from a Large-N station. The total data rate into the correlator for the Small-N case is $\sim k_{samp} \times k_{fs}/(k_{uni} \times k_{shad}) \sim k_{fs}/k_{uni} \sim \frac{1}{2}$ that of the Large-N case.

Table 4: Data Rate for One-Square-Degree Field

Small-N			Large-N		
B_{stn}	$=$	$F_{SN} \times k_{samp} \times B_{ele}$	B_{stn}	$=$	$F_{LN} \times k_{samp} \times k_{uni} \times B_{ele}$
	$=$	$280 \times k_{samp} \times B_{ele}$		$=$	$M \times k_{shad} \times k_{samp} \times k_{uni} \times B_{ele}$
B_{cor}	$=$	$N_{SN} \times k_{fs} \times B_{stn}$	B_{cor}	$=$	$(N_{LN}/M) \times B_{stn}$
	$=$	$N_{SN} \times F_{SN} \times k_{samp} \times k_{fs} \times B_{ele}$		$=$	$\frac{N_{LN}}{M} \times M \times k_{shad} \times k_{samp} \times k_{uni} \times B_{ele}$
	$=$	$32 \times 280 \times k_{samp} \times k_{fs} \times B_{ele}$		$=$	$N_{LN} \times k_{shad} \times k_{samp} \times k_{uni} \times B_{ele}$
	$=$	$9000 \times k_{samp} \times k_{fs} \times B_{ele}$		$=$	$8900 \times k_{shad} \times k_{samp} \times k_{uni} \times B_{ele}$

4.3 One-square-degree field-of-view

For both the Small- and Large-N cases, the stations send back multiple beams to the correlator to image the field-of-view. The sampling factor, k_{samp}, must be applied to both. The Small-N case also includes the foreshortening factor, k_{fs}. The Large-N case has the factors k_{shad} to account for the expansion of station baselines to avoid shadowing, and the factor k_{uni} to flatten the sensitivity across the field-of-view. Data rates from Small-N stations is $280/(M \times k_{shad} \times k_{uni}) \sim 1.4$ times that from Large-N stations. Since the Small-N array has far fewer stations, the total data rate into the correlator is nearly an order-of-magnitude less than for the Large-N array ($\sim k_{fs}/(k_{shad} \times k_{uni}) \sim 1/8$).

Note that this is anticipated to be the typical operating mode of the SKA: transmission of sufficient data to map a one-square-degree field-of-view. However, there is complete flexibility in transmitting only as many beams back as needed to map the field of interest, minimizing data transmission costs, in contrast to the Large-N configuration in the next section that always transmits at the maximum data rate.

4.4 Large-N without beamforming

In the previous section, the Large-N array data rate could be reduced if the output of *each* element is sent back to the correlator without any beamforming at the stations. Here the shadowing factor is not applied because cluster station-beams are not being formed; instead, each element is in effect a station, and the station beam is not affected by interelement spacings. The factor k_{uni} must be applied so that the Large-N map has uniform sensitivity. The data rate for Small-N stations is $\sim 280 \times k_{samp}/k_{uni} \sim 280$ times that for stations is a Large-N array. The total data rate into a Small-N correlator is $\sim k_{samp} \times k_{fs}/k_{uni} \sim 2$ times that going into a Large-N array correlator.

It is interesting to compare Large-N configurations with and without beamforming. Taking the ratio between the Large-N correlator data rates in Tables 4 and 5, we see that the case with beamforming is a factor of $\sim k_{samp} \times k_{shad} \sim 16$ times worse than a Large-N array with direct correlation. This is an important result for array design. If imaging of the full field-of-view is always required, then a large correlator is preferable to a system that feeds a smaller correlator

Table 5: Data Rate for One-Square-Degree Field with Large-N Without Beamforming

Small-N	Large-N
$B_{stn} = F_{SN} \times k_{samp} \times B_{ele}$ $= 280 \times k_{samp} \times B_{ele}$	$B_{stn} = k_{uni} \times B_{ele}$
$B_{cor} = N_{SN} \times B_{stn} \times k_{fs}$ $= N_{SN} \times F_{SN} \times k_{samp} \times k_{fs} \times B_{ele}$ $= 32 \times 280 \times k_{samp} \times k_{fs} \times B_{ele}$ $= 9000 \times k_{samp} \times k_{fs} \times B_{ele}$	$B_{cor} = N_{LN} \times B_{stn}$ $= N_{LN} \times k_{uni} \times B_{ele}$ $= 8900 \times k_{uni} \times B_{ele}$

through beamformers. However, the option of trading off field-of-view with data rate no longer exists in a system without beamformers.

5 Discussion and Conclusions

The results are summarized in Table 6. In the most general mode of operation, mapping a one-square-degree field, the Small-N concept has a slight disadvantage in data rates on the station-to-correlator link, but has a significant advantage in the total data into the correlator. This is because even though the Small-N configuration suffers a foreshortening loss, the losses for the Large-N concept (a diluted aperture to avoid shadowing and the need to equalize the sensitivity across the field) are more significant. The important points from this study are:

1. The data rate is proportional to the size of the field being mapped.
2. The data rate is proportional to the area of the station. Thus, Large-N stations will require lower-capacity links to the correlator than Small-N stations, but there will be more of these links.
3. Forming beams from arrays of small collectors incurs a penalty due to the dilution of the aperture that occurs when the small elements are spread out to avoid shadowing. This produces beams that are narrower than would be the case for filled apertures of the same effective area.
4. The noise in maps made by arrays of small apertures will increase moving from the centre of the field to the edge, while maps from large filled apertures (such as the LAR or phased arrays) will have uniform sensitivity. Thus for equivalent maps, additional observations are required by an array of small collectors to equalize the sensitivity across a map.
5. Large filled apertures suffer from a loss of sensitivity with increasing zenith angle due to foreshortening. To compensate, additional collecting area must be constructed, increasing the data rate into the correlator.

Table 6: Comparison of Data Rates for Different Array Configurations Using Ratio Small-N to Large-N

Configuration	From Station	Total Into Correlator
One station-beam	1	1/11
One Large-N station beam	5	1/2
One-square-degree	1.4	1/8
No Large-N beamformer	280	2

In these calculations the element bandwidth was kept constant. Another way to control the data rate is to reduce the signal bandwidth returned to the correlator. This solution could be applied equally well to both Large-N and Small-N arrays. The number of bits per sample could also be adjusted.

Unfortunately, we cannot yet take the next step and decide how to make tradeoffs between transmission bandwidth, correlator capacity, and field-of-view. We are prevented from doing this because the current specified field-of-view of the SKA is simply given as one square-degree at 1.4 GHz. Although it is commonly accepted that high-resolution observations utilizing continental-scale baselines will map a smaller field-of-view and thus require reduced data rates, this has yet to be written into a formal specification. Another reason for postponing tradeoff decisions is that costs and capabilities of fibre-optic transmission systems available in ten-years time (when the construction of the SKA begins) are unknown. The rate of technological progress in optical communications is out of our hands, but it would be highly beneficial to develop a more refined field-of-view specification to aid development of realistic SKA concepts.

References

[1] T. J. Cornwell. Radio-Interferometric Imaging of Very Large Objects. *Astronomy and Astrophysics*, 202:316–321, 1988.