SKA Memorandum # 24

Number of cross correlations required to synthesize a 1 square deg field of view

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Abstract

I consider the number of cross correlations required to synthesize a 1 deg^2 field of view (FoV) at 1.4 GHz in the context of different proposed SKA designs. It is shown that an array with 200m diameter elements plus large focal plane arrays requires almost an order of magnitude fewer cross correlations than an (optimized) array of 12m elements. It is also shown that for an array with a total of N elements arranged in stations with M elements per station, there is a value of M which minimizes the number of cross correlations required to synthesize the elemental FoV: $M_{min} = N^{1/2}$. Assuming 12m elements for the SKA, the minimum number of cross correlations required to image the elemental FoV (= 1 deg^2) is 8.3×10^5, and this is obtained when $M_{min} = 94$, arranged in 94 stations.

A key science driver for the SKA is a 1 deg^2 imaging FoV at 1.4 GHz. I consider the number of cross correlations required to synthesize such a FoV in the context of the proposed designs for the SKA.

One obvious method to synthesize a 1 deg^2 FoV is to perform a full cross correlation of signals from 12m antennas. The number of 12m diameter antennas in an SKA is: N = 8900. The number of cross correlations required is then:

$$X_{tot} = N(N-1)/2 = 4 \times 10^7 \qquad (1)$$

A second proposed design for the SKA using small elements entails phasing-up a number of elements, M, to form a 'station' beam, then cross correlating the station beams. I make the simplifying assumption that the station beams are a factor 1/M smaller in solid angle than those of the individual elements (ie. that the antennas are 'touching'). Hence the estimates below should be increased by a factor of order a few to take into account that some space is required between antennas.

The number of cross correlations per station required for station phasing $= M(M-1)/2$, and the number of stations, S, is: $S = N/M$. Hence the total number of cross correlations required to phase-up the stations, $X_{internal}$, is:

$$X_{internal} = S \times M(M-1)/2 \qquad (2)$$

The number of cross correlations per beam between the stations $= S(S-1)/2$, and the number of beams required to image the FoV of the individual elements (the 'elemental FoV') = M. Hence, the total number of cross correlations between stations, $X_{external}$, is:

$$X_{external} = M \times S(S-1)/2 \qquad (3)$$

Putting together the above relationships, the total number of cross correlations in this design is:

$$X_{tot} = M \times [S(S-1)/2] + S \times [M(M-1)/2] = \frac{N^2}{2M} + \frac{NM}{2} - N \qquad (4)$$

For example, using 12m elements, and assuming 16 elements per station leads to: $X_{tot} = 2.5 \times 10^6$.

Differentiating equ. (4) with respect to M gives: $M_{min} = N^{1/2}$ = the value of M that minimizes the number of cross correlations required to image the elemental FoV. In the case of 12m antennas for the SKA, the value of M that minimizes the number of cross correlations required to image the elemental FoV (= 1 deg^2) is: $M_{min} = 94$, arranged in 94 stations, and the minimum number of cross correlations is:

$$X_{tot,min} = 8.3 \times 10^5 \qquad (5)$$

Lastly, I consider the case of large collecting area antennas, such as the KARST or LAR design. Assuming 200m diameter elements requires $\mathrm{N} = 32$ antennas to achieve an SKA. The FoV of each element in this case is 0.0036 $\deg^2$, thereby requiring a focal plane array of 280 elements to image 1 $\deg^2$. The number of cross correlations per beam is: $\mathrm{N}(\mathrm{N}-1)/2 = 496$, and the total number of cross correlations required is then:

$$\mathrm{X}_{\mathrm{tot}} = 280 \times 496 = 1.4 \times 10^5 \qquad (6)$$

Again, there are factors of a few that have to be considered, due to station geometry, focal plane sampling requirements, etc..., but these factors will not change the basic conclusion that the large antenna + focal plane array design requires almost an order of magnitude fewer cross correlations than the small antenna design, even when the latter is optimized with respect to the number of elements per station. Of course, a fundamental requirement is that the optics and structure of the large antennas allow for large focal plane arrays.