# PULSAR SIGNAL PROCESSING ON UNIBOARD

Document number .............................................................. WP2-040.170.010-TD-001
Revision ............................................................................. 1
Author ............................................................................... A AhmedSaid
Date .................................................................................... 2011-04-01
Status .................................................................................. Approved for release

<table>
<thead>
<tr>
<th>Name</th>
<th>Designation</th>
<th>Affiliation</th>
<th>Date</th>
<th>Signature</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Additional Authors</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rob Ferdman, Ben Stappers</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Submitted by:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A AhmedSaid</td>
<td>UMAN</td>
<td></td>
<td>2011-04-01</td>
<td></td>
</tr>
<tr>
<td><strong>Approved by:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W. Turner</td>
<td>Signal Processing Domain Specialist</td>
<td>SPDO</td>
<td>2011-03-26</td>
<td></td>
</tr>
</tbody>
</table>
DOCUMENT HISTORY

<table>
<thead>
<tr>
<th>Revision</th>
<th>Date Of Issue</th>
<th>Engineering Change Number</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1st April 2011</td>
<td>-</td>
<td>First draft release for internal review</td>
</tr>
<tr>
<td>1</td>
<td>1st April 2011</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

DOCUMENT SOFTWARE

<table>
<thead>
<tr>
<th>Package</th>
<th>Version</th>
<th>Filename</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wordprocessor</td>
<td>MsWord</td>
<td>Word 2003 03m-wp2.040.170.010-td-001-1-pulsar SP on Uniboard-2003</td>
</tr>
<tr>
<td>Block diagrams</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ORGANISATION DETAILS

<table>
<thead>
<tr>
<th>Name</th>
<th>SKA Program Development Office</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physical/Postal Address</td>
<td>Jodrell Bank Centre for Astrophysics</td>
</tr>
<tr>
<td></td>
<td>Alan Turing Building</td>
</tr>
<tr>
<td></td>
<td>The University of Manchester</td>
</tr>
<tr>
<td></td>
<td>Oxford Road</td>
</tr>
<tr>
<td></td>
<td>Manchester, UK</td>
</tr>
<tr>
<td></td>
<td>M13 9PL</td>
</tr>
<tr>
<td>Fax.</td>
<td>+44 (0)161 275 4049</td>
</tr>
<tr>
<td>Website</td>
<td><a href="http://www.skatelescope.org">www.skatelescope.org</a></td>
</tr>
</tbody>
</table>
# TABLE OF CONTENTS

1 **INTRODUCTION** .................................................................................................................. 6
1.1 Purpose of the document ..................................................................................................... 6

2 **REFERENCES** ................................................................................................................... 7

3 **OVERVIEW** ....................................................................................................................... 8

4 **APPLICATIONS OF UNIBOARD TO RADIO PULSAR OBSERVING** .............................. 8

5 **DESIGN METHODOLOGY** ............................................................................................... 9
5.1 Algorithmic Simulation ....................................................................................................... 9
5.2 Architectural Modelling ...................................................................................................... 9
5.3 Hardware Implementation .................................................................................................. 9

6 **COHERENT PULSAR DEDISPERSION** ............................................................................ 10
6.1 Introduction ........................................................................................................................ 10
6.2 The FFT Method ............................................................................................................... 10
6.3 The Polyphase Filter Bank Method .................................................................................... 11

7 **FOLDING** ........................................................................................................................... 12
7.1 Mapping a dedispersion system into Uniboard ............................................................... 12

8 **PULSAR SEARCHING BY INCOHERENT DEDISPERSION** ............................................. 13
8.1 Motivation .......................................................................................................................... 13
8.2 Background ....................................................................................................................... 13
8.3 Search parameters ............................................................................................................. 14
8.4 Hardware requirements and computation time estimation .............................................. 17
8.5 Tradeoffs ........................................................................................................................... 20
8.6 Mapping a search system into Uniboard ........................................................................ 23
8.7 Summary and Looking Ahead ......................................................................................... 23
LIST OF FIGURES

Figure 1 The design methodology ........................................................................................................ 9
Figure 2 The FFT Dedispersion method ................................................................................................ 10
Figure 3: The Polyphase filter bank dedispersion method ..................................................................... 11
Figure 4 Pulsar data folding .................................................................................................................. 12
Figure 5 Incoherent dedispersion for 1 DM .......................................................................................... 14
Figure 6 Incoherent dedispersion architecture ..................................................................................... 15
Figure 7 4 inputs Taylor tree ................................................................................................................ 15
Figure 8 Rearranged 4 inputs Taylor tree .............................................................................................. 16
Figure 9 Radix 2 incoherent dedispersion architecture ....................................................................... 16
Figure 10 Radix 4 incoherent dedispersion architecture ...................................................................... 17
Figure 11 Parallel Pulsar Searching algorithm ....................................................................................... 17
Figure 12 Computation time Vs The total number of sub bands processed using the tree algorithm 22

LIST OF TABLES

No table of figures entries found.
LIST OF ABBREVIATIONS

DDR..............................Double Data Rate
DM...............................Dispersion Measure
FFT..............................Fast Fourier Transform
FPGA............................Field Programmable Gate Array
HDL..............................High Definition Language
IFFT.............................Inverse Fast Fourier Transform
ISM..............................Inter Stellar Medium
RAM..............................Random Access Memory
RTL.............................Register Transfer Level
SKA.............................Square Kilometre Array
TDD.............................Total DeDispersion time in clock cycles
TFFT.............................Total FFT computation time in clock cycles
1 Introduction

Possible methods for Pulsar signal processing including dedispersion and searching are considered within this document. These are used to provide estimates of hardware and computation time requirements.

An mapping of the functionality to the UNIBAORD FPGA processing platform is then performed to provide first estimates of cost and thermal dissipation.

1.1 Purpose of the document

This document is part of a series generated in support of the Signal Processing CoDR which includes the following:

- Signal Processing High Level Description [2]
- Technology Roadmap [3]
- Signal Processing Requirements [15]
- Signal Processing Costs
- Signal Processing Risk Register [16]
- Signal Processing Strategy to Proceed to the Next Phase [17]
- Signal Processing Co DR Review Plan
- Software & Firmware Strategy [18]
2 References

[4] Software Correlator Concept Description WP2-040.040.010-td-001 Rev A
[5] GSA Correlator Concept Description WP2-040.050.010-td-001 Rev A
[6] ASKAP Correlator Concept Description WP2-040.060.010-td-001 Rev A
[8] CASPER Correlator Concept Description WP2-040.080.010-td-001 Rev A
[9] ASIC-Based Correlator for Minimum Power Consumption-- Concept Description WP2-040.090.010-td-001 Rev B
[10] SKADS Processing WP2-040.100.010-td-001 Rev A
[12] Station Beamformer Concept WP2-040.120.010-td-001 Rev A
[16] Signal Processing Risk Register WP2-040.010.010.RE-001 Rev A
[17] Signal Processing Strategy to Proceed to the Next Phase WP2-040.010.030.PLA-001 Rev A
3 Overview

The UNIBOARD is a generic multi-FPGA board with high speed links intended for use in radio astronomy applications.

The goal of our project is to create FPGA architectures for pulsar applications such as real-time coherent pulsar dedispersion and folding and pulsar searching.

The main challenge that will be faced is processing large signal bandwidths of the order of hundreds of MHz to a few GHz in real time.

4 Applications of UNIBOARD to radio pulsar observing

The flexibility of the UNIBOARD FPGA hardware platform is well-suited to the many modes of pulsar observations and studies. Nearly all pulsar observations involve the removal of the dispersive effect of the interstellar medium (ISM) suffered by the incoming pulsar emission. To be more precise, the ISM acts as a phase filter that distorts the signal on the way to Earth by introducing dispersion. The degree of dispersion, i.e. the rate of frequency drift, depends on the integrated number of free electrons along the line of sight. This quantity is expressed as the “dispersion measure” (DM), and depends on the distance to the source, the observing frequency, and the bandwidth over which data is taken.

In order to account for, and ultimately remove, the deleterious effects of dispersion, two techniques can be applied: If the dispersion measure is known, the optimal procedure is the application of a corresponding inverse phase filter to the undetected voltages recorded with the telescope. This “coherent de-dispersion” completely removes these ISM effects and allows one to achieve the maximum time resolution, reproducing as faithfully as possible the signal originating at the pulsar being observed. Hence, coherent de-dispersion is essential for all high-precision timing applications. In the case where the DM is unknown (e.g. during the search for new pulsars), one has to split the observing bandwidth in to a large number (few hundreds to thousands for future applications) of narrow frequency channels, to search for the right frequency drift off-line in an “incoherent de-dispersion”, or “filterbank” technique.

In both cases, time sampling needs to be high. To perform coherent de-dispersion, Nyquist sampling of the analogue signal is necessary; in the incoherent case, sampling intervals of about 50 microseconds are desirable if one hopes to detect fast-spinning pulsars.

The need for both high time and frequency resolution for obtaining the highest quality data means that pulsar observations have always required the latest and best computational power and resources. In Europe, we currently have a range of pulsar backends, which are all trying to achieve the same end goal: coherent de-dispersion of data over as large a bandwidth as possible.

The most computationally expensive process in performing coherent de-dispersion requires the application of large FFTs to the undetected signal, multiplication with a correcting filter function, then finally performing inverse FFTs of this product. It has only become possible in the past decade to coherently de-disperse data over a substantial bandwidth, thanks to the increased performance, and decreasing cost, of off-the-shelf computing clusters, which perform these operations. In some
cases, it has been possible to do so in real time, depending on the required FFT lengths. Otherwise, the baseband data is recorded to disk for offline processing.

The real-time observation and coherent de-dispersion of pulsar data is becoming a necessity, as there are ongoing efforts towards obtaining larger and larger bandwidths of data and in the case of the SKA there will be multiple pulsars observed simultaneously, up to tens. The data rate required to obtain such volumes of raw data is incompatible with offline storage capabilities (and costs), whereas folded, de-dispersed pulse profile data (to be discussed later) is, by comparison, negligible in size. However, the ability of computer processors to perform real-time coherent de-dispersion in software amongst nodes on a computer cluster, or by using graphics processing units (which has recently seen a surge in popularity), is also becoming limited, due in part to disk input/output constraints. This is why the concept of UNIBOARD is a great match for the requirements of pulsar observations.

5 Design Methodology

![Fig. 1 The design methodology](image)

The adopted methodology consists of creating high level algorithm simulations, primarily within the MATLAB environment. These algorithms are then converted into accurate system models, with further simulation. Finally the models are converted into hardware implementations using an appropriate methodology.

Figure 1 illustrates this methodology.

5.1 Algorithmic Simulation

Using the Mathworks Matlab software, it is possible to simulate complex algorithms relatively quickly and easily. This algorithmic simulation allows the rapid evaluation of the different DSP methods that can be used and the exploration of the parameter space.

5.2 Architectural Modelling

The next step after the algorithmic simulation is the translation of the high level algorithms into system accurate Simulink models. This modelling stage takes into consideration the hardware architecture, its capabilities and limitations. For example fixed point integer arithmetic is used instead of floating point arithmetic.

5.3 Hardware Implementation

Finally, the models are converted into hardware using a combination of automatic netlist generation tools and HDL coding to structural and RTL level.
6 Coherent Pulsar Dedispersion

6.1 Introduction

The radiation from pulsars passes through the interstellar medium before reaching Earth. Due to the interstellar plasma, lower-frequency radio waves travel through the medium slower than higher-frequency radio waves. The resulting delay in the arrival of pulses at a range of frequencies is called ‘dispersion’. Dedispersion is the operation performed to try and invert the effects of dispersion.

The dispersion phenomenon affects the signal by delaying low frequencies more than the higher ones. The time delay between two frequencies $f_1$ and $f_2$ is calculated using the following formula:

$$T = D \cdot \left( \frac{1}{f_1^2} - \frac{1}{f_2^2} \right)$$

Where $D$ is a constant characteristic of a pulsar.

The time delays can be corrected in the frequency domain where they become phase shifts. These phase shifts can be calculated using the following formula:

$$PhaseShift(f) = e^{-j2\pi f \cdot T}$$

Where $f$ is the frequency, and $T$ the delay between $f$ and a reference frequency.

The reference frequency is chosen arbitrarily, for example it can be the lowest frequency, the highest frequency or the centre frequency of the received signal.

Two dedispersion methods are considered. The first one is used to de-disperse the full bandwidth of a signal at once. The second one performs the de-dispersion on smaller bandwidths. The following is a description of each method.

6.2 The FFT Method

![Figure 2 The FFT Dedispersion method](image-url)
This method, illustrated in Figure 2 consists of taking the signal into the frequency domain using an FFT, multiplying it with the dedispersion coefficients (phase shifts) then returning the signal into the time domain using an inverse FFT.

The FFT/IFFT lengths are relatively high and might range from 256M-1G points. For this reason, it is necessary to develop an FPGA architecture for large FFTs and IFFTs. Such architecture would use multiple ‘standard’ FFT cores to deal with these sizes and a sufficient amount of off chip memory. It is estimated that 4 FFT cores will be needed per large FFT/IFFT pair and for every 150-250MHz of bandwidth (depending on the FGPA clock). The off chip memory requirement in bytes is at least 12 times the size of the large FFT in order to store 2x3 blocks of data with 2 bytes per complex sample. The total bandwidth requirement is at least 12 times that of the FFT input data (1.8 GB/s-3 GB/s) because 6 complex sample need to be read/written every clock cycle.

6.3 The Polyphase Filter Bank Method

\[ X(n) \rightarrow \text{Polyphase Filter Bank} \rightarrow \text{Dedispersion} \rightarrow Y_i(n) \]

\[ \vdots \]

\[ \text{Dedispersion} \rightarrow Y_M(n) \]

*Figure 3: The Polyphase filter bank dedispersion method*

In this method, illustrated in Figure 3, the signal is split into a number of frequency channels using a polyphase filter bank, and then the FFT method is applied to de-disperse each channel.

The number of channels is likely to be in the range 64-256 channels of 4MHz bandwidth. With 4-8 coefficients per channel, the filter bank prototype filter will have 256-1024 coefficients and each coefficient will be represented by 8-12 bits.

In this method, the size of the FFTs in the dedispersion part of the processing is smaller than in the previous method. It is in fact divided by the number of channels. Therefore, the FFT length will be at most 4M points.
7 Folding

Folding is an operation where samples of the same pulse phase are accumulated together as illustrated in Figure 4. The pulse phase is predicted using a predetermined pulsar model. Folding is not a computationally intensive operation and has the advantage of reducing the data.

7.1 Mapping a dedispersion system into Uniboard

A Uniboard has 8 FPGAs and each one of them has access to two DDR3 banks of 2Gbytes each and capable of 800 M transfers/s of 64 bits. The FPGAs are arranged as two columns of 4 FPGAs, with one column used as an input column and the other one as an output column. The 4 input FPGAs are connected to 4 x 10GB/s links each, giving an input bandwidth per FPGA of 4Gbytes/s. This I/O bandwidth can allow signal bandwidths of up to 2GHz per input FPGA for a word length of 8 bits. Each memory bank will be sufficient to buffer up to 1 second of data for a signal bandwidth of 1GHz and a word length of 8 bits. Since there are two independent memory banks per FPGA, it will be possible to implement a double buffer to ensure a continuous flow of data. That assumes that the de-dispersion coefficients will be calculated on the fly by the FPGA and not stored in memory. However, it is believed that there are enough resources on the Uniboard FPGAs to do that in addition to having the polyphase filters and FFT engines.

If the input signals are divided into N frequency channels and if an M point FFT per channel is performed then it is assumed that it will take 2 x M x \log_2(M) cycles to compute an FFT and 4 x N x M x \log_2(M) cycles to compute N FFTs and N IFFTs. There is a factor of P reduction in computation time because P samples can be accessed simultaneously due to the high external memory bandwidth. For example on the Uniboard, the bandwidth of one memory bank is 64 x 800 MB/s and assuming an FPGA clock frequency of 200 MHz, 64 x 800/ (200 x Ws) samples (where Ws is the sample’s word length) can be accessed in one FPGA clock cycle. If B FPGAs are used to compute the FFT/IFFT, then the total FFT computation time \(T_{FFT}\) in clock cycles is:

\[ T_{FFT} = 4 \times N \times M \times \log_2(M)/(P \times B) \]
Let $T$ be the time duration of a buffer, $BW$ the signal’s bandwidth and $Ffpga$ the FPGA’s clock frequency. There will be $T \times BW = N \times M$ complex samples in a buffer and they have to be processed in no more than $T \times Ffpga$ cycles. Therefore, in order to be able to process data in real time:

$$4 \times T \times BW \times \log_2(T \times BW / N) / (P \times B) \leq T \times Ffpga \Rightarrow$$

$$BW \times \log_2(T \times BW / N) \leq Ffpga \times P \times B / 4$$

For example if $T = 1$ second, $Ws = 8$ bits and $B = 2$ then:

<table>
<thead>
<tr>
<th>$N$</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>16</th>
<th>32</th>
<th>64</th>
<th>128</th>
<th>256</th>
<th>512</th>
<th>1024</th>
<th>2048</th>
<th>4096</th>
</tr>
</thead>
<tbody>
<tr>
<td>$BW$ (MHz)</td>
<td>230</td>
<td>238</td>
<td>247</td>
<td>256</td>
<td>266</td>
<td>277</td>
<td>289</td>
<td>302</td>
<td>316</td>
<td>331</td>
<td>348</td>
<td>366</td>
<td>387</td>
</tr>
</tbody>
</table>

Since there are 4 rows of FPGAs on a Uniboard then the above bandwidths can be multiplied by 4 giving the following bandwidths for 1 Uniboard:

<table>
<thead>
<tr>
<th>$N$</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>16</th>
<th>32</th>
<th>64</th>
<th>128</th>
<th>256</th>
<th>512</th>
<th>1024</th>
<th>2048</th>
<th>4096</th>
</tr>
</thead>
<tbody>
<tr>
<td>$BW$ (MHz)</td>
<td>921</td>
<td>954</td>
<td>989</td>
<td>1026</td>
<td>1067</td>
<td>1110</td>
<td>1157</td>
<td>1209</td>
<td>1265</td>
<td>1326</td>
<td>1393</td>
<td>1467</td>
<td>1548</td>
</tr>
</tbody>
</table>

The above table suggests that 1 Uniboard will be able to coherently de-disperse a 1 GHz beam. If we were willing to accept a lower bit rate for the beams then it may be possible to process more bandwidth per pair of FPGAs.

### 8 Pulsar searching by incoherent dedispersion

#### 8.1 Motivation

We present below a schema for pulsar searching with boards containing FPGAs with large external RAM which we are planning to implement on the Uniboard architecture and which might be considered for use in SKA Phase 1. We describe the problem and the approach we have taken. We then discuss what sets of parameters would allow the proposed processing chain to fit on the Uniboard as it currently is laid out. We then look at whether this is a viable option for SKA Phase 1 based on known costs and projection of processing capabilities.

#### 8.2 Background

Pulsar searching involves de-dispersing signals for a range of trial Dispersion Measures (DMs) then looking for pulsar signatures in the obtained signals. The dedispersion is performed incoherently by splitting a signal into narrow frequency channels then time shifting each channel by an appropriate
time delay and finally adding the magnitudes of the samples from all the channels as illustrated in Figure 5

![Diagram of Incoherent dedispersion for 1 DM](image)

After dedispersion an FFT is applied to the whole data set captured over the observation period in order to look for periodicities characteristic of pulsars.

### 8.3 Search parameters

It is anticipated that the observation period will be about 10 minutes and the signal bandwidth 1 GHz (sampling at 2GHz). The number of frequency channels will be in the order of 8192 channels and the number of DMs will be at least 1000.

For these parameters, the total number of samples is 1200 GSamples which is reduced by a factor of 16 after the filter bank because the signals are down sampled by a factor of 8 by averaging 8 consecutive samples and an another reduction by a factor of 2 is due to the fact the input signals are real which means their spectrum is symmetric and only half of it needs to be kept. Although, after the channelization the samples become complex keeping the amount of data constant (with half the number of samples), the magnitude computation will reduce the data. The final total number of samples is 75 GSamples. If only 1 bit is used to store each sample then 8.7GB will required to store all the samples for the observation period.

Due to the amount of data and the nature of the processing, it will be necessary to buffer the data in a large external memory. This fact is very important because it influences directly the design of the architecture that can be used to carry out the task. Figure 6 shows what a possible pulsar searching system. It comprises three main parts: A filter bank, a dedispersion block and an FFT block, which are the required elements in a pulsar searching system. In addition, it comprises some data reorganisation blocks which might be needed to reorder the data in a way that is more suitable for the dedispersion or FFT blocks.
When performing the delay and add operations to dedisperse a signal for several DMs, there can be many operations that are common between DM trials. In order to exploit these redundancies a ‘Taylor tree’ algorithm has been described in the literature. This algorithm requires that the number of channels is equal to the number of DMs and that it is a power of 2. It allows the computation of \( N \) dedispersions of \( N \) channels using a number of operations proportional to \( N \log_2(N) \) rather than \( N^2 \).

Figure 6 Incoherent dedispersion architecture

![Incoherent dedispersion architecture](image)

Figure 7 4 inputs Taylor tree

Figure 7 illustrates a 4 inputs Taylor tree, where ‘\( T \)’ is a unit time delay. For each DM it is necessary to delay then add 4 signals. For 4 DMs, 16 operations are necessary but using a Taylor tree it is possible to perform the same task using just 8 operations.
The Taylor tree algorithm is similar in its structure to the FFT algorithm. A common basic structure in this algorithm is a *Delay-Add Butterfly* as highlighted in Figure 8. It is very similar to the FFT’s ‘Multiply-Add Butterfly’.

Inspired by FFT architectures, a possible incoherent dedispersion architecture is presented in Figure 9.

In this architecture, data is stored in the off chip memory, it is then loaded into internal input buffers, passed through a *Delay-Add Butterfly* and the result is stored in internal output buffer. From there, output data is stored back in the off chip memory and the process is repeated until all the operations are completed. The *Delay-Add Butterfly* consists of two adders and a ‘d’ samples delay. The delay ‘d’ could be equal to 1 or whatever is deemed appropriate. The delay ‘d’ can also be variable to allow the generalisation of the Taylor tree algorithm which assumes unit delays and is only applicable to small bandwidths where the dispersion curve is linear. By having a variable delay it might be possible to apply the Taylor tree algorithm to large bandwidths if a good schedule of delay-add operations can be found (using a computer program).
If the off chip memory bandwidth is high enough then multiple butterflies can be used. It is also possible to use a higher radix butterfly such as a radix 4 butterfly as illustrated in Figure 10. However, in this case the number of channels and DMs must be a power of 4.

### 8.4 Hardware requirements and computation time estimation

Let $N_t$ be the total number of channels and $M$ the number of samples per channel. The $N_t$ channels are divided into $B$ bands of $N$ channels, i.e., $N_t = B \times N$. The bands are processed in parallel independently from each other.

For each band, dedispersion is performed for the same DMs. After dedispersion, signals from all bands are added together for each DM to produce the final result.
The Taylor tree algorithm is used to de-disperse a band. In order to be able to use this algorithm a band is divided into small enough sub bands to satisfy the conditions of the algorithm.

Let \( L \) be the number of sub bands and \( K \) the number of channels per sub band, i.e, \( N = L \times K \). The tree algorithm is applied to each sub band to compute \( K \) DMs then the de-dispersed signals from the \( L \) sub bands are accumulated to produce \( K \) DMs for all the band \((N \text{ channels})\). The process is repeated until all the DMs are processed.

To process \( K \) DMs, \( M \times K \times \log_2(K) \) samples are read from memory then written back to memory. To complete the processing of all the sub bands \( L \) times more read and write operations are performed, i.e, \( M \times N \times \log_2(K) \). If a total of \( D \times K \) DMs are wanted then this will cost \( D \times M \times N \times \log_2(K) \) read and write operations. The accumulation of de-dispersed signals from each sub band requires reading \( D \times M \times N \) samples and writing \( D \times M \times K \).

Therefore, in total, processing one band requires reading \( D \times M \times N \times (1+ \log_2(K)) \) samples and writing \( D \times M \times (K + N \times \log_2(K)) \) samples.

\[
\text{Total number of samples to read} = D \times M \times N \times (1+ \log_2(K))
\]

\[
\text{Total number of samples to write} = D \times M \times (K + N \times \log_2(K))
\]

The number of samples to store input data is \( N \times M \). It is also necessary to store a copy of data for computing \( K \) DMs and another to store the final result. In total \( 2 \times N \times M + D \times K \times M/B \) samples need to be stored per band.

\[
\text{Total number of samples to store per band} = 2 \times N \times M + D \times K \times M/B
\]

It is very likely that the external memory will be able to deliver several samples per FPGA clock cycle, because the memory clock will very likely be a multiple of the FPGA clock, the memory will provide a double data rate (for DDR) and the memory data width will be a multiple of a sample’s data width. The number of samples \( P \) that can be accessed per FPGA clock cycle is equal to the external memory bandwidth \( BW_{mem} \) divided by the product of the sample’s data width \( Ws \) by the FPGA clock frequency \( F_{fpga} \).

\[
P = BW_{mem} / (W_s \times F_{fpga})
\]
To take advantage of the capabilities of the external memory, data has to be organised in matrix form where the rows represent sub bands and the columns channels. Each raw would store data of all channels for one sub band. The data of each channel is stored consecutively and is followed by the data of the next channel. If the number of sub bands is higher than the maximum number of rows, then other rows can be stored consecutively by increasing the number of columns. Using this storage scheme, \( P \) samples can be accessed every FPGA clock cycle which allows the use of \( P \) parallel butterfly units to accelerate the processing.

The number of FPGA clock cycles needed to complete the processing of one band \( NFPGACC \) is equal to sum of the total number of samples to read and the total number of samples to write (neglecting the latency of a butterfly) divided by \( P \). FPGA cycles to process 1 band:

\[
NFPGACC = \frac{(D \times M \times N \times (1 + \log_2(K)) + D \times M \times (K + N \times \log_2(K)))}{P}
\]

After processing each band in parallel, the de-dispersed signals must be accumulated for each DM. This requires gathering \( B \times D \times K \times M \) samples processing them and storing \( D \times K \times M \) samples. Gathering these samples from the different parallel systems will take more time than simply reading data from memory but for the sake of estimation it is approximated as being equal to the memory access time. Therefore the final total de-dispersion computation time in clock cycles, \( TDD \) can be expressed as:

\[
TDD = D \times M \times (N \times (1 + \log_2(K)) + (K + N \times \log_2(K)) + (2B+1) \times K)/P \quad \Rightarrow
\]

\[
TDD = D \times M \times (Nt \times (1 + 2 \times \log_2(K)) / B + 2 \times (B+1) \times K) \times (Ws \times Fpga) / BWmem
\]

The final total memory storage requirement is:

\[
TotalExtMemSize = (B \times 2 \times N \times M + D \times K \times M) \times Ws \quad \Rightarrow
\]

\[
TotalExtMemSize = (2 \times Nt \times M + D \times K \times M) \times Ws
\]
If the total number of DMs $D \times K = Nt$ then:

\[
\text{TotalExtMemSize} = 3 \times Nt \times M \times Ws
\]

The next step in pulsar searching is to perform the FFT of all the signals dispersed at the different DMs. Each signal has $M$ samples and it is assumed that it will take $2 \times M \times \log_2(M)$ cycles to compute its FFT. The number of DMs is $D \times K$ but they are divided into $B$ sets that can be processed in parallel. There is also a factor of $P$ reduction in computation time due to the parallelisation permitted by the external memory bandwidth. This leads to a total FFT computation time $T_{FFT}$ in clock cycles of:

\[
T_{FFT} = 2 \times Nt \times M \times \log_2(M)/(P \times B) \quad =>
\]

\[
T_{FFT} = 2 \times Nt \times M \times \log_2(M) \times (Ws \times Fpga) / (BW_{mem} \times B)
\]

The FFT computation does not require any additional memory storage.

For the FPGA resources, it is estimated that $P$ de-dispersion butterfly units and $P$ FFT butterfly units can be fitted very easily ($P$ is expected to be $\leq 1024$). Therefore, it is believed that de-dispersion and FFT operations can be performed on the same FPGA. The internal memory requirement will probably not be a problem because the size/depth of the memory is arbitrary and can be reduced if necessary. However, it is believed that it will not be possible to connect many parallel memory modules to one FPGA only, therefore it will be necessary to use one FPGA per band.

### 8.5 Tradeoffs

In order to be able to implement the searching algorithm and run it in ‘real time’, two conditions must be satisfied:

1. The required hardware must be affordable;
2. The computation time must be faster than or equal to the observation time.

The following is a summary of the formulas that we have developed above to estimate the memory size and the computation times:

\[
\text{TotalExtMemSize} = (2 \times Nt \times M + D \times K \times M) \times Ws
\]
\[ TDD = D \times M \times (Nt \times (1+2 \times \log2(K)) / B + 2 \times (B+1) \times K) \times (Ws \times Ffpga) / BW_{mem} \]

\[ TFFT = 2 \times Nt \times M \times \log2(M) \times (Ws \times Ffpga) / (BW_{mem} \times B) \]

These formulas are the basis of the following analysis:

The hardware affordability is mainly influenced by the amount of external memory that will be required to store and process the input data. Reducing the memory size can be achieved by using the lowest possible number of bits to store the data (Ws). This will also reduce the computation time. If reducing Ws is not enough then it will be necessary to reduce the number of DMs to compute, that will reduce \( D \times K \), or reduce the signal's bandwidth, which will reduce \( Nt \times M \).

The computation time is not influenced in the same manner as the memory size by the different parameters. The presence of the \( \log2(K) \) factor suggests that it is highly desirable to increase \( K \). The impact of increasing \( K \) will be even greater if the number of DMs was equal to the total number of channels \( Nt \). In this case \( D \times K = Nt \), which means increasing \( K \) will also reduce \( D \) for each of the subbands.

Figure 12 shows the computation time as a function of \( D \) in the case where \( D \times K = Nt \). The values of the other parameters are as follows:

- \( ObservationTime = 600 \) sec;
- \( SamplingFrequency = 2 \)GHz;
- \( Nt = 8192 \);
- \( M = (ObservationTime \times SamplingFrequency/16)/Nt \);
- \( B = 8 \);
- \( Ffpga = 200\)MHz;
- \( BW_{mem} = 2 \times 64 \times 800 \);
- \( Ws = 8 \) bits.

As it can be seen if \( D > 32 \), i.e. \( K < 256 \), it will not be possible to perform real time pulsar searching in this case. The total memory requirement is 210 GB or 26.2 GB per FPGA which might be too high.
Figure 12 Computation time Vs The total number of sub bands processed using the tree algorithm

The best case scenario is when $D = 1$ and $K = N$. In this case, the de-dispersion computation time is minimised and more importantly, there is no need to have extra storage for intermediate computations. The input data can be overwritten because it will not be needed anymore therefore the memory size requirement drops to:

$$\text{TotalExtMemSize} = Nt \times M \times Ws$$

And the computation times becomes:

$$TDD = M \times (Nt \times (1+2 \times \log_2(Nt/B)) / B + 2 \times (B+1) \times Nt/B) \times (Ws \times Ffpga) / BWmem$$

$$TFFT = 2 \times Nt/B \times M \times \log_2(M) \times (Ws \times Ffpga) / (BWmem \times B)$$

Now the memory size is divided by 3 and assuming the same values for the other parameters as above, the memory requirement is 8.73 GB per FPGA which is more likely to be feasible and affordable.
8.6 Mapping a search system into Uniboard

A Uniboard has 8 FPGAs and each one of them has access to two DDR3 banks of 2GB each capable of 800 M transfers/s of 64 bits.

The parameters values that have been used in the previous section match specifically the Uniboard case. Therefore, for \( D \times K = Nt \) with \( D < 64 \) and:

- \( \text{ObservationTime} = 600 \text{ sec} \);
- \( \text{SamplingFrequency} = 2\text{GHz} \);
- \( Nt = 8192 \);
- \( M = 9.16E6 \);
- \( F_{fpga} = 200\text{MHz} \);
- \( Ws = 8 \text{bits} \).

The memory required per FPGA is 26.2 GB which is higher than the available 4 GB. In the second case where \( D = 1 \) and \( K = N \), 8.73 GB per FPGA are required. If \( Ws \) is reduced to 4 bits then 4.4 GB per FPGA will be required and with a 10 % reduction of the signal bandwidth 4 GB per FPGA will be required. Thus, using one Uniboard it is possible to implement a real time pulsar search engine for a bandwidth of 900 MHz and 1024 DMs.

Another possibility is to use several UNIBOARDS, for example 4 boards, with \( Ws = 4 \), would be able to process 1 GHz and 8192 DMs.

8.7 Summary and Looking Ahead

We have shown that it is likely possible to coherently dedisperse a full 1 GHz of dual-polarisation bandwidth, appropriately divided into manageable subbands, with a single Uniboard. The number of subbands required results in a bandwidth per subband which is still large enough to achieve the time resolution desired for the high precision timing experiments. Based on today’s prices a Uniboard in production will cost about 10,000 euros. Given that in SKA phase one we are looking at trying to time a few tens of pulsars simultaneously this should be achievable for something on the order of 250,000 euros. The expected power consumption of 350 W per board would then result in a total of about 8 kW. Each board would likely need a control PC and the data would also need to be archived.

In the case of the pulsar searching the current Uniboard could likely deal with the expected bandwidth required for the Galactic plane search at a central frequency of about 800 MHz. The bandwidth for that search is likely to be less than that assumed for the calculations above (based more on the 1-2 GHz range) and by adjusting the number of bits required we could make it possible to dedisperse and search about 500 MHz of bandwidth. A more detailed analysis of the requirements for the Aperture Array search would be required, but the total number of dispersion trials is likely to be similar to the above, and the total data rate less, so it will probably also fit.

Assuming 1 GHz beams and a price per Uniboard of 10,000 euros per beam, and considering the number of dispersion trials we can estimate then it will cost about 5 euros per beam per DM. This metric is useful as one of the limiting factors is the number of dispersion measures which needs to be calculated. As above assuming 350 W power consumption per board then it will cost 0.17 W per beam per DM. However, these estimations are specific to Uniboard and should be considered as upper limits because it is possible that the Uniboard resources and the FPGAs will not be fully utilised.
It is estimated that something like 4000 beams will be needed in SKA Phase 1 and in this model one Uniboard is needed for each beam. However it may be possible that the increased processing capabilities, and hopefully increased amount of memory on Uniboard 2, or a subsequent board, will improve the efficiency and reduce cost. As noted above it is possible that the capability of the fpgas in Uniboard will not be fully utilised and so a next step would be to consider how we might be able to implement the computationally costly acceleration search as part of this package.