Figure of Merit for SKA survey speed

John D. Bunton, *CSIRO ICT Centre*

9 September 2003

Introduction

Survey speeds for the SKA concepts vary considerably due to large differences in field of view and bandwidth. The differences in bandwidth make it difficult to directly compare the concepts and here a simple figure of merit is proposed. This figure of merit is proportional to the time needed to cover a given area to a given sensitivity and takes into account different observing strategies that may be adopted to reach this sensitivity.

Figures of Merit

Survey speed depends on sensitivity (S), here sensitivity normalised to the SKA specification of A/Tsys = 20,000. An instrument with half the sensitivity will need four times as much integration time per field to achieve the same sensitivity, reducing its survey speed by a factor of four. Survey speed also depends on directly on the imaging field of view (FOV) and on the correlator bandwidth (BW). Thus a simple measure of survey speed (FBproduct) is the product of the FOV and the BW, weighted by the sensitivity squared.

$$\text{FBproduct} = \text{FOV} \times \text{BW} \times S^2$$

However the fractional bandwidths for the SKA can be very large, for example an HI survey at redshifts up to z=3 would cover 0.35 to 1.4 GHz. A survey of a contiguous area of sky over this range of frequencies leads to the situation that at low frequencies each pixel is imaged many times because the fields of view now overlap for field centres set by the highest frequency. For most SKA concepts the FOV at 0.35 GHz is 16 times larger than at 1.4 GHz. This leads to the equivalent of a 16 times increase in integration time, and therefore increased sensitivity at the lower frequency (LAR and FAST are exceptions because their focal plane area is limited). When the same survey is done with a design concept using a narrow band correlator, the survey will be conducted sequentially over a number of frequency bands. If the same antenna pointings are used for each frequency band then the narrow band system will achieve the same relative sensitivity at all frequencies as the wide band system. The time taken by the narrow band system to do the survey is longer by a factor equal to the number of bands. In this case the field of view – bandwidth product (FBproduct) accurately characterises the survey speed.

For narrow band systems an alternative strategy is keep the overlap between fields constant by increasing the distance between field centres as the observed frequency decreases. This decreases the number of fields needed to cover a given area as the observed frequency decreases. In the case given above, the lowest observing band at 0.35 GHz will take 1/16 of the time needed for highest observing band at 1.4 GHz. With this strategy the integration time and sensitivity remains constant as a function of frequency but sensitivity is sacrifice compared to the wide band implementation. The advantage of this strategy is that it significantly increases survey speed for the narrow band system while still meeting a minimum sensitivity specification.

As a hypothetical example consider a survey that is to observe all frequencies from 0 Hz to a maximum frequency (f_{max}) and an instrument that can observe this frequency range instantaneously. Assume this hypothetical instrument takes one unit of time to do the observation. Now consider a second instrument that can observe half the frequency range instantaneously. For an equal sensitivity observation it needs one unit of time to observe the frequency band f_{max} to $f_{max}/2$ and one quarter of a unit time to observe the band from $f_{max}/2$ to 0 Hz. The instrument with half the bandwidth takes 1.25 units of time to do a constant sensitivity observation. In the table below the relative time needed to cover all frequencies up to a given maximum frequency is tabulated for instrument with different observing bandwidths. The fractional bandwidth is the ratio of the instantaneously observable bandwidth (BW) to the maximum frequency. To observe all frequencies from f_{max} to 0 Hz requires a time sequence of observations in a number of bands. The number of bands that must be observed to complete the survey is the reciprocal of the fractional bandwidth.

Table 1 Relative survey time for wideband survey as a function of number of frequency bands

No. of bands to cover all frequencies	Fractional Bandwidth (BW/f_{max})	Relative time needed to complete survey	Time reduction (TR)
1	1	1	1
2	.5	1.25	0.625
4	.25	1.875	0.469
8	.125	3.1875	0.398
N (large)	1/N	N/3	0.3333

The time reduction (TR) is equal to the relative time needed to complete the survey divided by the number of bands. As a function of fractional bandwidth (fb) the reduction in time relative to that needed when equal time is allocated to each frequency band is

$$TR(fb) = 1/3 + fb/2 + fb^2/6.$$

Where fb is equal to the bandwidth (BW) divided by the maximum frequency (f_{max}) observed in the survey. This equation was derived for the case where the full bandwidth is covered by an integer number of bands. It is also representative of other bandwidths. Consider the case where the fractional bandwidth equals 0.75. In this case one and one third bands are needed to cover all frequencies and the total time needed is 1.065 units. Using the time reduction factor the time needed is estimated as 1.3333*TR(.75) = 1.069 units, which is sufficiently accurate for our purposes.

A reduction in time to do a survey results in an increase in survey speed. Thus a survey speed figure of merit that compensates for the differences in bandwidth between the different concepts is

$$\mathrm{FOM} = \mathrm{FOV} \times \mathrm{BW} \times S^2 / \mathrm{TR}(\mathrm{BW}/f_{max})$$

For BW equal to f_{max} this figure of merit is equal to FBproduct. In effect the survey FOM uses the fastest possible survey, one that covers all frequencies up to the maximum frequency instantaneously, as the reference. As BW decreases, and all other parameters are left unchanged, the new survey FOM also decreases but in less than a directly linear fashion. This is shown in the figure below. The new figure of merit can be interpreted as quantifying the survey speed where a constant integration time is guaranteed for each field and frequency band observed. If Tsys were constant with frequency this results in a survey which minimises time for a given sensitivity.

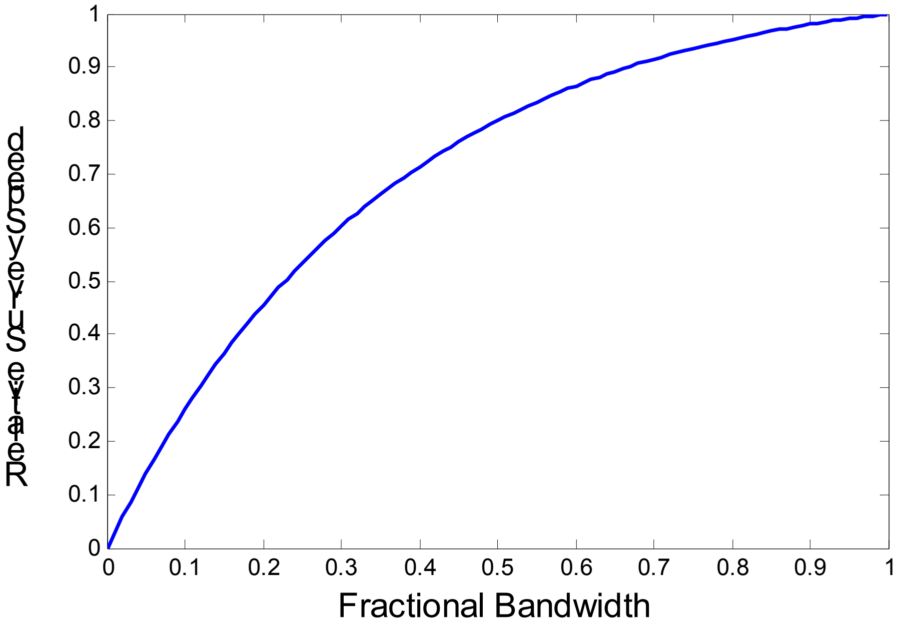

Figure 1 Relative survey speed as a function of fractional bandwidth

Survey FOM for SKA Concepts

The survey figures of merit at 1.4 GHz for the current SKA concepts and the standard SKA specifications are listed in the table below. Both 12m reflector concepts [1, 2] are grouped together under the same heading and it assumed that both have the same correlator performance as the LNSD concept [1]. These concepts and the LAR concept [3] have correlator bandwidth far in excess of 1.4 GHz and for comparison purposes it is assumed that the frequencies from 0.15 to 1.4 GHz are imaged simultaneously. In the update to the KARST proposal [5] a focal plane array up to 0.5 square degrees is considered possible. However, no bandwidth information is given. Here a 3:1 bandwidth or 930 MHz is assumed. The field of view of LAR and KARST is limited by the focal plane array and the field of view does not increase as frequency decreases. This means that for these concepts there is no increase in survey speed as frequency decreases. Thus the survey FOM is equal to the FBproduct for these two concepts.

For the aperture array data from the 2003 update [4] is used. It specifies that 44 out of 70 200MHz beams are needed to image one square degree. Thus the imaging FOV is 1.6 square degrees at 1.4 GHz with an imaging bandwidth of 200 MHz. At 1.4 GHz the relative sensitivity is 0.55, increasing to 1 at 670 MHz. To compensate for this increase in sensitivity the integration time below 670 MHz should be decreased by a factor of 3.3. From 1.4 GHz to 670 MHz there are 3.65 200-MHz bands and observing these with constant sensitivity takes 2.45 units of time. The next two bands, 670-470 and 470-270 MHz, take 0.1 units of time, and from 270 MHz down the sensitivity doubles again and the time taken is negligible. Thus compared to a full bandwidth system the aperture array concept is slower by a factor of 2.55. Thus its survey FOM is calculated to be 1.6 degrees * 1400 MHz * 0.55^2 / 2.55 = 265. As expected this value is a little larger than the value of 237, which is calculated using the equation for survey FOM and ignoring the increased sensitivity at low frequencies.

Table 2: Survey figures of merit for a 1.4GHz SKA

Concept	Sensitivity S	Imaging FOV, deg^2	Bandwidth BW, MHz	FBproduct deg^2.MHz	FOM deg^2.MHz	Relative speed
Cylinder	1	48	800	38,400	57018	48.51
12 m	1	1.5	1250	1,875	2055	1.75
LAR	1	1	1250	1,250	1250	1.06
SKA spec	1	1	780	780	1175	1.00
KARST	1	0.5	1050	525	525	0.45
Luneburg lens Update	0.5	0.76 12	750 46	143 138	220 394	0.19 0.34
Luneburg lens Original	1	0.138 3.74	750 46	104 80	159 234	0.14 0.20
Aperture Array	0.55	1.6	200	97	265	0.23

Both the cylinder and Luneburg lens proposal can trade bandwidth for FOV allowing the full power of the correlator to be used in a survey. For the cylinder almost the full antenna FOV could be processed but here the data for a 48 deg^2 field of view is used [6]. For the Luneburg lens the survey field of view has been improved by a factor of ~5 in the update [7] when compared to the original concept [8]. However the update has increased the number of feed arms and reduced the total collecting area by a factor of two. This reduces the relative sensitivity S to 0.5. For comparison purposes data for the original concept [8] is also given. The increase in FOV is largely offset by the decrease in sensitivity.

In the last column of the table the FOMs are compared to the value for the standard SKA specifications of 1 square degree with a bandwidth of 0.5 + f/5 GHz. It is seen that most concepts come close to reaching this performance. Both the Luneburg Lens and aperture array could significantly increase their speed by increasing their data transmission and correlator performance. An aperture array sharing infrastructure with a high frequency concept would provide a large increase in survey speed. Alternatively a change from 8-bit data to 4-bit data would double the survey speed for both concepts without any increase in cost. Another alternative for the Luneburg lens is to install only one feed arm. This would increase the relative sensitivity. But each antenna station would double in area decreasing the beam size by a factor of two. The net effect of this change would be a doubling in survey speed without any significant change in cost. With 4-bit data and a single feed arm, the Luneburg Lens survey FOM would be 1580 bringing it close to the performance of the 12m concepts.

Neither LAR nor FAST can significantly increase their survey speed because the FOV is limited by the size of the focal area and the bandwidth is already large. For the 12m concepts the use of a focal plane array could increase the FOV. By trading bandwidth for FOV the correlator of the 12m concept could handle a three times increase in FOV at 1.4 GHz, but beyond that the correlator cost would increase in direct proportion to the FOV.

The relative speed of surveys with the cylinder is an order of magnitude greater at 1.4GHz than the other concepts. Two factors explain the speed, a filled aperture antenna station and a FOV-BW tradeoff. The FOV that can be imaged by an SKA correlator of a given capacity is directly proportional to the area of the aperture that correlator inputs are derived from (for arrays of separate antennas other factors are also involved). A prime example of this is LAR which needs very little correlator capacity. Compared to the 12m concepts the cylinder has a filled aperture that is about fifteen times greater, making the correlator fifteen times more efficient at imaging.

The second factor is the FOV - bandwidth tradeoff. This is possible because at full bandwidth the antenna FOV is very much greater than the FOV that is imaged. Thus the bandwidth allocated to a single beam can be partitioned between multiple beams allowing an increase in the number of beams and the FOV at the expense of bandwidth. In the example used the bandwidth is reduced from the full 4.9 GHz to 800 MHz allowing six times as many beams to be processed. These factors are partly offset by the higher Tsys of the cylindrical concept. Taken together the cylindrical reflector concept can image about 22 times (15 times greater area x 6 times as many beams / 4 for a doubling in Tsys) as much sky as the 12m concepts for the same correlator capacity. This factor is closely matches the ratio of the FBproduct for the two concepts. See the appendix for a more detailed analysis of the difference in survey speed between the 12m LNSD concept and the cylindrical reflector concept.

The aperture array concept does not operate much above 1.4GHz and the KARST concept is currently limited to 6 GHz. The comparison below includes KARST and the other reflector and refractor concepts at 6 GHz. No data on a 6-GHz focal plane FOV for KARST is available so a scaling from the 1.4 GHz system is given. At 6 GHz the full bandwidth of the LAR, KARST and 12m correlators can be used. This increases the relative performance of these concepts. The relative performance of the Luneburg lens and the cylindrical reflector decrease because there is no increase in correlator capacity and the bandwidth of the SKA specification has increased. The cylinder FOM is now ten times greater than that of the 12m concepts.

Table 3: Survey figures of merit for a 6 GHz SKA

	Sensitivity S	Imaging FOV, deg^2	Bandwidth MHz	FBproduct deg^2.MHz	FOM deg^2.MHz	Relative speed
Cylinder	1	1.300	1600	2,080	4347	22.94
12 m	1	0.082	3200	261	404	2.13
LAR	1	0.054	4000	218	218	1.15
SKA Spec.	1	0.054	1700	93	190	1.00
KARST	1	0.027	4500	123	123	0.65
Luneburg lens	0.5	0.041	750	8	19	0.10
Update	0.5	0.653	46	8	22	0.12
Luneburg lens	1	0.008	750	6	14	0.07
Original	1	0.119	46	5	16	0.09

Only the cylinder, 12m and LAR concept are specified to operate at 20 GHz. However the Luneburg lens could also possibly operate at this frequency and has been included (no scaling for sensitivity loss has been included). The FBproduct is the same as for the 6 GHz figure of merit but reduced by the decrease in FOV. The relative speed for all concepts has decreased because the bandwidth for the SKA specification has increased. In all cases the fractional bandwidth is less than 0.2 which increases the survey FOMs considerably when compared to the Fbproducts, except for LAR. Because of this LAR's relative speed has decreased the most.

Table 4: Survey figures of merit for a 20 GHz SKA

	Sensitivity S	Imaging FOV, deg^2	Bandwidth MHz	FBproduct deg^2.MHz	FOM deg^2.MHz	Relative speed
Cylinder	1	0.1200	1600	192	513	10.57
12 m	1	0.0074	3200	24	56	1.16
LAR	1	0.0049	4000	20	20	0.40
SKA Spec.	1	0.0049	4500	22	49	1.00
Luneburg lens	0.5	0.0037	750	0.7	2	0.04
Update	0.5	0.0588	46	0.7	2	0.04
Luneburg lens	1	0.0007	750	0.5	1	0.03
Original	1	0.0107	46	0.5	1	0.03

Conclusion

The survey speed of the SKA is approximately proportional to FOV, BW and the square of the sensitivity. For equal sensitivity, the product of these must be divided by a time reduction factor that compensates for the higher speed of narrow band systems at low frequencies.

At 1.4 GHz the 12m concepts and LAR concepts and the standard SKA specification have similar survey speeds. KARST, the aperture array and the Luneburg Lens are just a little slower. For the aperture array and the Luneburg lens significant increases in speed are possible. The cylindrical reflector concept has the highest speed, being more than an order of magnitude faster due to the large FOV of the antenna and the filled aperture design. At higher frequencies the LAR and 12m concepts can better utilise their correlators and the performance advantage of the cylindrical concept is reduced to a factor of ten.

Acknowledgement

The author would like to thank Mike Kesteven for his critical reading of this paper.

Bibliography

All SKA white papers are found at the international SKA web site
http://www.skatelescope.org

[1] USSKA Consortium, "The Square Kilometer Array Preliminary Strawman Design Large N - Small D" SKA Design concept white paper 2, July 2002

[2] Swarup, G "Preloaded Parabolic Dish Antennas for the Square Kilometer Array" SKA Design concept white paper 4, July 2002

[3] The LAR Group, "The Large Adaptive Reflector Concept" SKA Design concept white paper 5, July 2002

[4] The Aperture Array approach for the SKA, Editors: A. van Ardenne, H. R. Butcher, SKA Design concept white paper 14, May 2003

[5] Nan et al, "Kilometer-square Area Radio Telescope - KARST", SKA Design concept white paper 1, July 2002

[6] Panorama of the Universe: A Cylindrical Reflector SKA, Editor J.D. Bunton, SKA Design concept white paper 13, May 2003

[7] Eyes on the Sky: A Refracting Concentrator Approach to the SKA, Editor A. Chippendale, Design concept white paper 10, May 2003

[8]" Eyes on the Sky: A Refracting Concentrator Approach to the SKA" Editor Peter Hall, SKA Design concept white paper 6, July 2002

[9] Bunton, J.D., Jackson, C.A and Sadler, E.M., 'Cylindrical Reflector SKA', SKA Design concept white paper 7, July 2002

Appendix: Comparison of 12m and cylindrical correlator limited survey speed

The correlators for the 12m LNSD concept and the cylindrical reflector concept are very similar in size. The 12m LNSD concept has 4400 antennas [1] and the cylinder has 600. When operated as an instrument that processes circular beams each cylindrical reflector is broken up into 8 subsections (2002 white paper [9]). To process these circular beams an 8 x 600 = 4800 input correlator is needed which 20% larger than that needed by the 12m LNSD concept. Note, that at 1.4 GHz the eight circular beams can be placed anywhere within the 120^o x 1^o antenna field of view available with the updated cylindrical concept [6].

For the cylinder there is considerable freedom in how the signals from the line feed are aggregated. To maximise the imaged FOV the best strategy is to beamform along the full length of the line feed and generate fanbeams. At 1.4 GHz these fanbeams have an area of 1/8 of a square degree. The correlator needed to process one fanbeam now has 600 inputs, one from each cylindrical reflector. This correlator forms 179,700 baselines as compared to 11,517,600 when circular beams are processed. The capacity needed by the fanbeam correlator is 1/64th that of the circular beam correlator. Thus the circular beam correlator has the capacity to process 64 fanbeams simultaneously. These fanbeams have a total field of view that is eight times larger than that of the original circular beam and can be placed anywhere within 120^o by 1^o antenna field of view.

Generating pencil beams is an option for the 12m LNSD concept but in aggregating N antennas the field of view does not decrease by N but by N times the filling factor. Although filling factors of 30% are indicated in the 2002 white paper, a more realistic figure, which allows unblocked observing at low elevations, is 10%. Thus for a grouping of 12 antennas the field of view is reduced by a factor of 120 compared to the field of view for a single antenna. For the same correlator capacity the LNSD concept can process 144 of these pencil beams leading to a small increase in field of view. The advantage in forming pencil beams is largely negated by the fact that the group of 12 antennas form an aperture that is not filled. In addition an extra feed or focal plane array is needed to make use of the additional field of view that pencil beam allow.

Assuming pencil beams are not used with the LNSD proposal then the cylindrical reflector is 8 times more efficient. However the LNSD concept has circular field of view 1.5 times greater than that of the cylindrical reflector. This reduces the relative performance benefit of the cylindrical reflector correlator to 5. The cylindrical reflector correlator is also larger, it process 20% more baselines and its bandwidth is 50% larger. Adding these two factors gives the cylindrical reflector concept a nine times higher survey speed, as is seen at 20GHz. At lower frequencies the lower fractional bandwidth possible with the cylindrical reflector concept gives it a further 35% advantage at 1.4 GHz. Also at 1.4 GHz all the correlator capacity can be used. With the current LNSD concept only 1.25 GHz out of 3.2 GHz is used leading to a further factor of 2.56 to the advantage of the cylindrical reflector concept. With these two factors (1.35 and 2.56) included the cylindrical reflector survey speed is some 30 times greater than that of the current 12m LNSD concept at 1.4 GHz.