# FIGURES OF MERIT FOR SKA CONFIGURATION ANALYSIS

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# ORGANISATION DETAILS

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# LIST OF ABREVIATIONS

EMI ........ Electromagnetic Interference
RFI ........ Radio Frequency Interference
RQZ ........ Radio Quiet Zone
SKA ........ Square Kilometre Array
SPDO ........ SKA Program Development Office

# 1 Introduction

This note summarises the Figures of Merit that will be applied in gauging the site specific configurations for the SKA. These figures of merit can be applied to the Phase 1 or Phase 2 configurations and are intended to capture science related properties and a measure of the risk of elements being affected by electromagnetic interference (EMI).

It is noted that the analysis of the configurations addressed here does not include simulations of array performance on (artificial) sources. Such investigations are underway and will be reported separately.

# 2 PSFRMS

The PSFRMS figure of merit was introduced by Tim Cornwell in [1]. It is a measure of the expected side-lobe level in the synthesized beam. Applying Parseval's theorem, by assessing the variation in filling factor in uv cells, one obtains a measure of the rms side-lobe level in the synthesized beam. The following steps are taken to obtain the "PSFRMS" quantity from a sample of uv-points:

1. Divide the uv-plane into cells, sufficiently small to over-sample the synthesized beam in the spatial domain (i.e. cell width = dish diameter / 2.4, so the number of cells in the uv plane is given by (2.4 x maximum baseline length / dish diameter)$^2$. For example, the uv plane for a layout of 15m dishes with a maximum baseline of 2,000 km would have 320,000 cells on a side, giving a total of 1011 uv cells.
2. Into these uv cells place the uv points. Let the number of uv points be $N_{uv}$. For typical SKA configurations out to ~3,000km baselines, the vast majority of these uv cells will be empty, however, near to the core there may be a large number of uv cells that contain more than one uv point. Let the number of uv points in uv cell i,j be denoted $n_{ij}$.
3. Following ALMA Memo 18, find the standard deviation of the occupancy ($n_{ij}$) of the uv cells, this is the "PSFRMS" figure of merit value:

$$PSFRMS = \sqrt{\sum_{i,j} n_{ij}^{2}} \Big/ N_{uv}$$

4. When we assess configurations we take only one sample every ten minutes, for computational load reasons, which is not a realistic SKA scenario. This means that the PSFRMS value is not a good approximation to the actual power in the synthesized beam, but in the context of comparing different configurations the PSFRMS is still minimised if the number of cells sampled in the uv-plane is maximised for fixed $N_{uv}$ (so that all the $n_{ij}$ are unity or zero). So it remains a useful relative measure.

# 3 UVGAP

The general uv-gap algorithm is presented in [5]. A new version of the algorithm has been used, which differs from the general algorithm in that there is no binning of uv-points in the |u| direction. The approach is as follows:

1. Calculate the uv-points for a given layout and observation set -up (again, as with the PSFRMS, we use 1 sample every 10 minutes). Assign each uv point a value of $q= (u^2+ v^2)^{1/2}$ and polar angle, $\theta$ (in the range 0-π), using conjugate symmetry to re-map points that initially lie between π and 2π. (This is equivalent to generating both the (u,v) and (-u, -v) points and only using half the plane.)
2. Bin these $q$, $\theta$ points in $\theta$, with the bin size in $\theta$ chosen to correspond to the sampling time – for example a sampling time of 10 minutes would require 72 bins in 12 hours, corresponding to a bin width of π/72 radians or 2.5 degrees.
3. Now in each angular bin sort the uv points into increasing value of q. A simple example of such an arrangement is shown in Figure 1.
4. Calculate the unbinned uv-gap value from $q$=0 out to a $q$ value of $q_{max}$. This might correspond to some baseline range that we are interested in such as 3 km, 30 km or 3,000 km, for example.
5. Let there be N points in the angular bin which are have $q$ values less than $q_{max}$ . Calculate the fractional change in $q$ for each of these N points: for the k<sup>th</sup> point, $\partial q_k= \Delta q_k/q_k = (q_k-q_{(k-1)})/q_k$.
   Then calculate the integral average of $\partial q$ by assigning $\partial q(q_{k-1}, q_k)\equiv \partial q_k$.
   We take only the uv points that have $q < q_{max}$, so if $q_N$ is the last *uv*-point within a given sector we take $q_{N+1} = q_{max}$ and $\delta q_{N+1}\equiv 1$. Then, at the low $q$ end of the samples, we assign $q_0 = 0$. In other words, for the portions of the $q$ wedge that are before the first and after the last uv point, the $\delta q$ value is set to unity. The integrated mean in the *i*th angular wedge is then calculated for $\delta q$ between $q = 0$ and $q = q_{max}$.
6. The average over all angular bins of the $\langle\delta q\rangle_i$ values is then taken to generate a single number for the uvgap value of an array, for a particular value of $q_{max}$ and for a given observational set up.
7. So, to give the example fully, here we would calculate the uvgap value in the example as:

$$\text{uvgap}_{\text{example}} = \frac{1}{q_{max}}\sum_{k=1}^{7}\partial q_k \times (q_k - q_{k-1}), \text{ with } q_0=0 \text{ and } q_7 = q_{max},$$
$$= q_1/q_{max} + (q_2-q_1)^2/(q_2q_{max}) + (q_3-q_2)^2/(q_3q_{max}) + (q_4-q_3)^2/(q_4q_{max}) + (q_5-q_4)^2/(q_5q_{max}) + (q_6-q_5)^2/(q_6q_{max}) + (q_{max}-q_6)/ q_{max}.$$

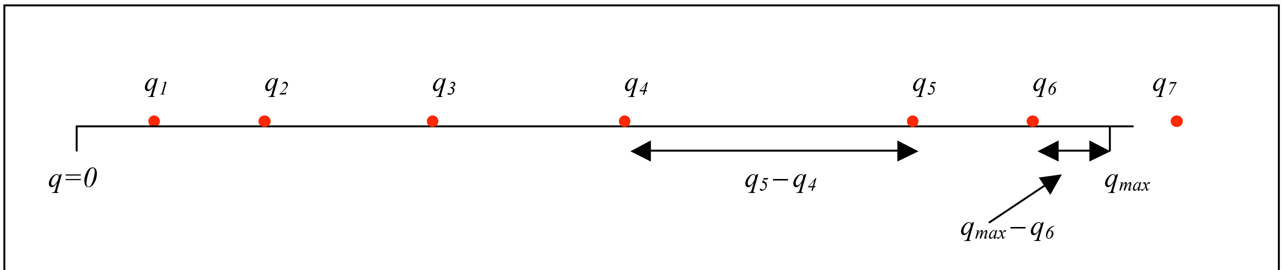


**Figure 1: A simple example of some uv points within an angular bin.**

# 4 EMI risk

To allow configurations to be designed in areas that are blocked by applying the EMI buffer zones around farmsteads, a method has been developed for treating those particular buffer zones not as no-go areas, but as EMI risks, where the risk increases closer to the farmstead, see [4].

For each collector the individual EMI Risk Figure is found by assessing the distance from the antenna to all the farmsteads. If the distance to all farmsteads is greater than the original limit of **10.5km** then this antenna has an EMI risk of zero. If the antenna is a distance less than **10.5km** from one farmstead location then the EMI risk for this antenna is non-zero. The original limit results from analysis of expected levels of EMI emanating from a farmstead, attenuated by propagation, such that the maximum level at the receptor equals RA769-2 level + 15 dB (which applies to connected-element interferometers).

We need to scale the EMI risk number for each antenna so that it ranges from zero at the limit (**10.5km**) to unity at some inner limit ($r_0$). For this inner limit it can be argued that the level of harmful interference for VLBI telescopes is taken, i.e. RA769-2 + 40 dB, or 25 dB higher. In [4] it is concluded that the appropriate scaling with radius goes as $r^{-3.7}$. This then leads to a value of $r_0$ of **2.2km**.

It is possible that a dish may be less than **10.5km** from more than one farmstead, in which case we add the risk figures from each farmstead in quadrature, assuming that these fields are uncorrelated. For dish *i*, a distance $r_j$ from the $j^{th}$ farmstead, the dish EMI risk figure is then given by:

$$EMI_i = \sqrt{\sum_{r_j < r_1} \left[ \left( r_j^{\ e} - r_1^{\ e} \right) \Big/ \left( r_0^{\ e} - r_1^{\ e} \right) \right]^2}$$

where $r_0$ is the inner limit (where, and within which, the EMI risk figure from an individual farmstead to an individual dish is set to unity), $r_1$ is the outer limit beyond which the EMI risk figure falls to zero (this is **10.5km**) and *e* is the EMI field scaling index, -3.7.

Once the EMI risk figure has been calculated for each collector in the array the information is displayed in several ways:

1. The global average value of this EMI risk figure is calculated, averaging over all collectors of the same type. This will give a single value for each collector array. A value of zero would mean that no collectors are within the **10.5km** limit. A value of unity could mean that all dishes are less than $r_0$ from one farmstead.
2. For each collector type plots will be provided of the binned maximum, median, $25^{th}$ percentile and $75^{th}$ percentile values of the individual collector EMI risk figures, with bins in radial distance from the array centre, probably logarithmically spaced so that they contain similar numbers collectors. This will give an idea of the range of values.
3. A histogram for the whole array will be provided, showing the number (or fraction) of collectors that have EMI risk figures in various bins.
4. A histogram showing the closest distance that each collector is from a farm will be produced, and any collectors closer than the VLBI limit of 2.2km from a farm will be noted.

Examples are found in [4].

# 5 Sky Visibility

The Sky Visibility Figure of Merit was one of the metrics used in the short-listing process, see [6]. The visible sky for any given array will depend on the extremes of the east-west and north-south distribution of receptors in the array and will be a function of declination. The masks that were generated for the two configurations used a specification of 15 degrees minimum elevation, instead of 30 degrees used at the time of short-listing. Therefore it is appropriate to use 15 degrees in the calculations for the sky visibility. Sources must be above the minimum elevation as seen from all receptors simultaneously for at least 4 hours in a 24 hour period.

# 6 UV Coverage

Plots of uv coverage will be generated for a range of cases, in a similar way to site short-listing, see [6]. The cases that will be evaluated and presented are summarised in the Appendix, section 9.

# 7 Beam Shape

Plots of synthesized beam shape will be generated for a range of cases, in a similar way to site short-listing, see [6]. The cases that will be evaluated and presented are summarised in the Appendix, section 9.

# 8 References

[1] Quality indicators for the MM array, T. Cornwell, ALMA memo 18, 5-7-1984
[2] The algorithms used to generate the array Figures of Merit "PSFRMS" and "uvgap", R.C. Bolton, A. Lobanov, R.P. Millenaar, 5 November 2009
[3] Figures of merit for SKA scale configurations, R.C. Bolton, A. Scaife, G. Grigorescu, R.P. Millenaar, A. Lobanov, Widefield Science and Technology for the SKA, SKADS Conference 2009
[4] Proposal for a Configurations EMI Risk Figure, V1.0, R.C. Bolton, R.P.Millenaar, 23-6-2010
[5] Array configuration studies for the Square Kilometre Array – Implementation of figures of merit based on spatial dynamic range, D.V. Lal, A.P. Lobanov, S. Jiménez-Monferrer, SKA Memo 107, November 2009
[6] SKA array configuration studies – a guide for SKA site proposers, CSTF, 18-5-2005

# 9 Appendix: Evaluation parameters

Metric for testing site-specific layouts in the tool used for analysis: iAntConfig.
Use "observations" for each layout combining all source declinations (3 options), all HA ranges (3 options) and all channel numbers (2 options) – 18 combinations in total. Four layout types for each site, 8 layouts in total.

| | | | | |
|---|---|---|---|---|
| Dish layout: Mid and inner | 2400 15 m dishes within 180 km from centre | | | |
| Shadowing diameter | 15 m | | | |
| Uv cell size | 6.25 m | | | |
| Elevation limit | 15 degrees | | | |
| Source declination | +1.3 | -28.7 | -58.7 | (degrees) |
| HA Range | 0 (snapshot) | 0-2 hrs | 0-4 hrs | Sampling every 10 mins |
| Centre frequency | 1.4 GHz | | | |
| Fractional bandwidth | 0.25 | | | |
| Number of channels | 1 | 10 | | |
| Baseline range for test | Min 20 m | Max 200 km | | |

| | | | | |
|---|---|---|---|---|
| Dish layout: incl. remote | 243 groups of dishes (25 remote stations with 24 dishes plus 218 groups of 11 dishes) | | | |
| Shadowing diameter | 15 m | | | |
| Uv cell size | 145.8 m | (equivalent to 350 m station diameter) | | |
| Elevation limit | 15 degrees | | | |
| Source declination | +1.3 | -28.7 | -58.7 | (degrees) |
| HA Range | 0 (snapshot) | 0-2 hrs | 0-4 hrs | Sampling every 10 mins |
| Centre frequency | 1.4 GHz | | | |
| Fractional bandwidth | 0.25 | | | |
| Number of channels | 1 | 10 | | |
| Baseline range for test | Min 20 km | largest in config. | | |

| | | | | |
|---|---|---|---|---|
| AA1 layout | 250 stations of 60 m diameter | | | |
| Shadowing diameter | 1 m (i.e. no shadowing) | | | |
| Uv cell size | 25 m | (Equivalent to 60 m station diameter) "Oversampling" set to 1/25 = 0.04 | | |
| Elevation limit | 45 degrees (NB 4hrs HA is always below 45 degrees elevation) | | | |
| Source declination | +1.3 | -28.7 | -58.7 | (degrees) |
| HA Range | 0 (snapshot) | 0-2 hrs | 0-4 hrs | Sampling every 10 mins |
| Centre frequency | 650 MHz | | | |
| Fractional bandwidth | 0.25 | AA-mid frequency range 400-1450MHz | | |
| Number of channels | 1 | 10 | | |
| Baseline range for test | Min 20 m | Max 200 km | | |

| | | | | |
|---|---|---|---|---|
| AA2 layout | 250 stations of 180 m diameter | | | |
| Shadowing diameter | 1m (i.e. no shadowing) | | | |
| Uv cell size | 75m | (Equivalent to 180m station diameter) "Oversampling" set to 1/75 = 0.01333 | | |
| Elevation limit | 45 degrees | | | |
| Source declination | +1.3 | -28.7 | -58.7 | (degrees) |
| HA Range | 0 (snapshot) | 0-2 hrs | 0-4 hrs | Sampling every 10 mins |
| Centre frequency | 260 MHz | | | |
| Fractional bandwidth | 1.46 | AA-low fractional BW e.g. 70-450 MHz , centre at 260, range =380 MHz | | |
| Number of channels | 1 | 10 | | |
| Baseline range for test | Min 20 m | Max 200 km | | |